The "Free Will Theorem"

of Conway & Kochen

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Introduction. In a recent paper,[1] John Conway and Simon Kochen claim to have proven that "if the choice of a particular type of spin one experiment is not a function of the information accessible to the experimenters [*i.e.*, is an unforced decision made by exercise of their free will], then its outcome is equally not a function of the information accessible to the particles." I have found their argument to be elusively slippery. I write here in an effort to place the essentials of their argument in sharper focus.

A remarkable property of solitary spin-one particles. The spin matrices of a spin-one particle can be taken to be

$$\left.\begin{aligned} \mathbb{S}_1 &= \hbar\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \\ \mathbb{S}_2 &= \hbar\begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix} \\ \mathbb{S}_3 &= \hbar\begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{aligned}\right\} \tag{1}$$

which are seen to satisfy the required commutation relations

$$\begin{aligned} [\mathbb{S}_1, \mathbb{S}_2] &= i\hbar\mathbb{S}_3 \\ [\mathbb{S}_2, \mathbb{S}_3] &= i\hbar\mathbb{S}_1 \\ [\mathbb{S}_3, \mathbb{S}_1] &= i\hbar\mathbb{S}_2 \end{aligned}$$

[1] Foundations of Physics **36**, 1441–1473 (2006).

From the form of the sum of their squares

$$\mathbb{S}^2 = \mathbb{S}_1^2 + \mathbb{S}_2^2 + \mathbb{S}_3^2 = 2\hbar^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

it is evident that $[\mathbb{S}_k, \mathbb{S}^2] = \mathbb{O}$ $(k = 1, 2, 3)$. Almost as obviously, their squares

$$\mathbb{S}_1^2 = \hbar^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$\mathbb{S}_2^2 = \hbar^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$\mathbb{S}_3^2 = \hbar^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

commute with each other, so comprise a compatable set of observables. Each has eigenvalues $\{0, \hbar^2, \hbar^2\}$, while $\mathbb{S}^2$ has eigenvalues $\{2\hbar^2, 2\hbar^2, \hbar^2\}$. The striking implication is that if a state ρ (whether pure or mixed) is presented to $\mathbb{S}_i$, the resulting eigenstate presented to $\mathbb{S}_j$, and the then resulting eigenstate presented finally to $\mathbb{S}_k$ (here i, j, k are any permutation of $1, 2, 3$), the meter readings $\{\mathcal{M}_i, \mathcal{M}_j, \mathcal{M}_k\}$ are certain to present some permutation of $\{1, 0, 1\}$,[2] and the last meter reading is always forced, redundant—a foregone conclusion.

This is a straightforward implication of orthodox quantum theory—the upshot of what Conway & Kuchen call the SPIN axiom. I emphasize that it is special to the case of spin one. For spin one-half one has

$$\mathbb{S}_1 = \tfrac{1}{2}\hbar \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \mathbb{S}_2 = \tfrac{1}{2}\hbar \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \mathbb{S}_3 = \tfrac{1}{2}\hbar \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Again, the squares commute, but now for an uninteresting reason:

$$\mathbb{S}_1^2 = \mathbb{S}_2^2 = \mathbb{S}_2^2 = \tfrac{1}{4}\hbar^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Such measurements do not tell us anything, and they conform trivially to the fact that $\mathbb{S}^2$ measurements always read $\frac{3}{4}\hbar^2$. For spin three-halves one has

[2] The meters are calibrated in units of $\hbar^2$, which I henceforth set equal to unity.

$$\mathbb{S}_1 = \tfrac{1}{2}\hbar\begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}$$
$$\mathbb{S}_2 = \tfrac{1}{2}\hbar\begin{pmatrix} 0 & -\sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & -2 & 0 \\ 0 & 2 & 0 & -\sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}$$
$$\mathbb{S}_3 = \tfrac{1}{2}\hbar\begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}$$

and finds that the squares of these matrices fail to commute; indeed, one has

$$[\mathbb{S}_1^2, \mathbb{S}_2^2] = [\mathbb{S}_2^2, \mathbb{S}_3^2] = [\mathbb{S}_3^2, \mathbb{S}_1^2] = \hbar^2\begin{pmatrix} 0 & 0 & \sqrt{3} & 0 \\ 0 & 0 & 0 & -\sqrt{3} \\ -\sqrt{3} & 0 & 0 & 0 \\ 0 & \sqrt{3} & 0 & 0 \end{pmatrix}$$

So the game is stopped bfore it can even get under way. And so (presumably) it goes for all higher spin values. All subsequent remarks pertain exclusively to the case spin = one.

Instruments that measure spin in the direction indicated by the unit vector $\boldsymbol{a}$ are represented by the matrix

$$\mathbb{S}(\boldsymbol{a}) \equiv a_1\mathbb{S}_1 + a_2\mathbb{S}_2 + a_3\mathbb{S}_3$$

and measurement of spin-squared in the direction $\boldsymbol{a}$ is represented by

$$\mathbb{S}^2(\boldsymbol{a}) \equiv \big\{a_1\mathbb{S}_1 + a_2\mathbb{S}_2 + a_3\mathbb{S}_3\big\}^2 = \mathbb{I}_3 - \begin{pmatrix} a_1a_1 & a_1a_2 & a_1a_3 \\ a_2a_1 & a_2a_2 & a_2a_3 \\ a_3a_1 & a_3a_2 & a_3a_3 \end{pmatrix}$$

(in which I have again set $\hbar = 1$). Thought of as an operator on real 3-space, this matrix acts by projection onto the plane $\perp \boldsymbol{a}$ (which accounts for its spectrum: $\{0, 1, 1\}$), but by intention it will act on the complex 3-space of spin states.

Peres' simplification of the Kochen-Specker no-go theorem. Einstein, who embraced a philosophy of *local determinism*, would interpret the 101 rule as an indiction that within every solitary spin-one particle lurks an "instruction set" that tells the particle how to respond to spin-squared measurements. To $\mathbb{S}^2(\boldsymbol{a})$-measurement the instruction set might say "announce 0," else it will say "announce 1." In the former case, it will for every $\mathbb{S}^2(\boldsymbol{b})$-measurement ($\boldsymbol{b} \perp \boldsymbol{a}$) instruct "announce 1." In the latter case it will, for every orthogonal pair $\{\boldsymbol{b}_1, \boldsymbol{b}_2\}$ in the $\boldsymbol{b}$-plane, instruct "announce 1 (else 0)" if/when subjected to $\mathbb{S}^2(\boldsymbol{b}_1)$-measurement, and will instruct "announce 0 (else 1)" if/when subjected to $\mathbb{S}^2(\boldsymbol{b}_2)$-measurement. Einstein, Podolsky and Rosen argued (1935) that so long as such instruction sets were ignored the "quantum-mechanical description of physical reality" must be considered incomplete. Nevertheless ...

Simon Kochen and Ernst Specker (1967), by an intricate argument the essence of which had been anticipated by Andrew Gleason (1957), managed to demonstrate that no instruction sets with the properties just stated cannot exist: the stipulated properties present a combinatorial problem that possess no solution. Asher Peres (1991) devised the much simpler variant of the Kochen-Specker argument of which Conway & Kochen make use.

Peres is led by the following figure to as set of 33 rays (directions in 3-space) which can be used construct a population of 72 orthogonal pairs, and a population of 16 orthogonal triads. The instruction set can be interpreted to

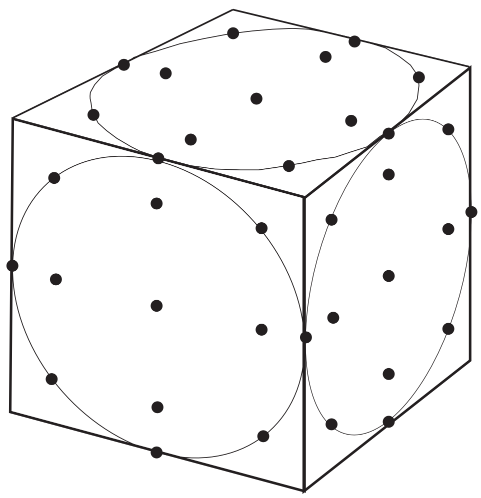

FIGURE 1: *The cube from which Peres obtains his set of 33 rays.*

require that one and only one leg of each of the 16 triads be painted red. This can be accomplished in many ways. But in every case at least one of the 72 orthogonal pairs ends up with both members red—which is disallowed.

J. S. Bell (1964) had shown that if one places a pair of spin-one-half particles in a certain entangled state (the singlet state) and looks to certain correlational features of the data generated when such a system is subjected to certain measurements, and if one finds those correlational features to be in violation of a certain inequality, then one cannot attribute those correlational features to instruction sets hidden within the particles. That the inequality in question is in fact violated was established by convincing experiment: "spooky action-at-a-distance" was shown thus to be a fact of nature; quantum mechanics was shown to be profoundly non-local.

But the non-existence of hidden variables was, by this line of argument, supported only by *statistical* evidence. Kochen-Specker's argument—the first of what has by now become a large population of "no-go theorems"—has in this the light the great merit of being non-statistical: it establisheds that if one grants the correctness of the 101 rule (which follows from the barest rudiments of quantum theory, and is anyway subject to direct verification) then instruction sets of the sort previously contemplated become a *logical* impossibility.

Instruction sets of the sort previously contemplated are said to be "**non-contextual**" because they are formulated ray-by-individual-ray: they tell the particle how to respond to $\mathbb{S}^2(\boldsymbol{a})$-measurement, independently of any other measurements to which it might be subjected. The Kochen-Specker does not serve to exclude the possibility of "**contextual**" instruction sets which would, in the present instance, be formulated triad-by-individual-triad: they would tell the particle how to respond to $\{\mathbb{S}^2(\boldsymbol{a}), \mathbb{S}^2(\boldsymbol{b}), \mathbb{S}^2(\boldsymbol{c})\}$, where the response to $\mathbb{S}^2(\boldsymbol{a})$ might vary from triad to triad. To address this possibility, Conway & Kochen bring into play...

A remarkable property of entangled spin-one particle pairs. Consider a composite system assembled from two spin-one particles, of which one is accessible to Alice, the other to Bob, whose lab is some distance away. Angular momentum operators *for the composite system* are

$$\mathbb{SS}_k = \mathbb{S}_k \otimes \mathbb{I} + \mathbb{I} \otimes \mathbb{S}_k \quad : \quad k = 1, 2, 3$$

where the 3×3 matrices $\mathbb{S}_k$ were defined at (1). Working from

$$\mathbb{SS}_i \mathbb{SS}_j = \mathbb{S}_i \mathbb{S}_j \otimes \mathbb{I} + \mathbb{S}_i \otimes \mathbb{S}_j + \mathbb{S}_j \otimes \mathbb{S}_i + \mathbb{I} \otimes \mathbb{S}_i \mathbb{S}_j \tag{2}$$

we find that these 9×9 matrices satisfy

$$\begin{aligned}
[\mathbb{SS}_1, \mathbb{SS}_2] &= i\hbar \mathbb{SS}_3 \\
[\mathbb{SS}_2, \mathbb{SS}_3] &= i\hbar \mathbb{SS}_1 \\
[\mathbb{SS}_3, \mathbb{SS}_1] &= i\hbar \mathbb{SS}_2
\end{aligned}$$

From (2) it follows also that

$$\mathbb{SS}_k^2 = \mathbb{S}_k^2 \otimes \mathbb{I} + 2\,\mathbb{S}_k \otimes \mathbb{S}_k + \mathbb{I} \otimes \mathbb{S}_k^2$$

$$\mathbb{SS}^2 \equiv \mathbb{S}^2 \otimes \mathbb{I} + 2\sum_{k=1}^{3} \mathbb{S}_k \otimes \mathbb{S}_k + \mathbb{I} \otimes \mathbb{S}^2$$

$$[\mathbb{SS}^2, \mathbb{SS}_1] = [\mathbb{SS}^2, \mathbb{SS}_2] = [\mathbb{SS}^3, \mathbb{SS}_3] = \mathbb{O}_9$$

The eigenvalues of $\mathbb{SS}^2$ are[3] $\{0, 2, 2, 2, 6, 6, 6, 6, 6\}$, while those of $\mathbb{SS}_k$ $(k = 1, 2, 3)$ are $\{-2, -1, -1, 0, 0, 0, +1, +1, +2\}$. The singlet state is defined by the

[3] I again set $\hbar = 1$.

equations

$$\mathbb{SS}^2|\text{singlet}) = \mathbb{SS}_k|\text{singlet}) = 0\cdot|\text{singlet}) \quad : \quad k=1,2,3$$

which give

$$|\text{singlet}) = \tfrac{1}{\sqrt{3}}\begin{pmatrix}1\\0\\0\\0\\1\\0\\0\\0\\1\end{pmatrix}$$

In terms of the eigenstates of $\mathbb{S}_3$—which are

$$\boldsymbol{u} = \tfrac{1}{\sqrt{2}}\begin{pmatrix}-i\\1\\0\end{pmatrix}, \quad \boldsymbol{n} = \begin{pmatrix}0\\0\\1\end{pmatrix}, \quad \boldsymbol{d} = \tfrac{1}{\sqrt{2}}\begin{pmatrix}+i\\1\\0\end{pmatrix}$$

and satisfy

$$\begin{aligned}\mathbb{S}_3\,\boldsymbol{u} &= +\boldsymbol{u}\\ \mathbb{S}_3\,\boldsymbol{n} &= \ \ 0\,\boldsymbol{n}\\ \mathbb{S}_3\,\boldsymbol{d} &= -\boldsymbol{d}\end{aligned}$$

—one has

$$|\text{singlet}) = \frac{\boldsymbol{u}\otimes\boldsymbol{d}+\boldsymbol{n}\otimes\boldsymbol{n}+\boldsymbol{d}\otimes\boldsymbol{u}}{\sqrt{3}}$$

which shows the singlet state to be a manifestly entangled state of the composite system.

Alice is equipped with a meter—represented

$$\mathbb{A}(\boldsymbol{a}) = (a_1\mathbb{S}_1 + a_2\mathbb{S}_2 + a_3\mathbb{S}_3)^2\otimes\mathbb{I}$$

—that, upon interaction with her particle, measures "spin squared in the direction defined by the unit vector $\boldsymbol{a}$." The meter is calibrated to read either 0 or 1, since the eigenvalues of $\mathbb{A}(\boldsymbol{a})$ are $\{0,0,0,1,1,1,1,1,1\}$. The spectral representation of $\mathbb{A}(\boldsymbol{a})$ is immediate:

$$\mathbb{A}(\boldsymbol{a}) = 0\cdot\mathbb{A}_0(\boldsymbol{a}) + 1\cdot\mathbb{A}_1(\boldsymbol{a})$$

where $\mathbb{A}_1(\boldsymbol{a})\equiv\mathbb{A}(\boldsymbol{a})$ projects onto a 6-space, and $\mathbb{A}_0(\boldsymbol{a})\equiv\mathbb{I}-\mathbb{A}(\boldsymbol{a})$ projects onto the complementary 3-space. Bob is similarly equipped, with a meter represented

$$\mathbb{B}(\boldsymbol{b}) = \mathbb{I}\otimes(b_1\mathbb{S}_1 + b_2\mathbb{S}_2 + b_3\mathbb{S}_3)^2$$

Consider now the following sequence of events: the composite system is initially in the singlet state, represented by the density matrix

$$\rho = |\text{singlet})(\text{singlet}|$$

Alice makes a measurement. Her meter reads 0 with probability

$$\text{tr}[\,\mathbb{A}_0(\boldsymbol{a})\rho\,] = \tfrac{1}{3} \quad \text{and prepares the state} \quad \rho_0 = \frac{\mathbb{A}_0(\boldsymbol{a})\rho\mathbb{A}_0(\boldsymbol{a})}{\text{tr}[\,\mathbb{A}_0(\boldsymbol{a})\rho\mathbb{A}_0(\boldsymbol{a})}$$

Else her meter reads 1 with probability

$$\text{tr}[\,\mathbb{A}_1(\boldsymbol{a})\rho\,] = \tfrac{2}{3} \quad \text{and prepares the state} \quad \rho_1 = \frac{\mathbb{A}_1(\boldsymbol{a})\rho\mathbb{A}_1(\boldsymbol{a})}{\text{tr}[\,\mathbb{A}_1(\boldsymbol{a})\rho\mathbb{A}_1(\boldsymbol{a})}$$

Now Bob does a spin-squared measurement on his particle, getting 0 or 1 with probabilties that are conditioned by the result of Alice's prior measurement (*i.e.*, by the system-state that Alice's measurement prepared), and are given by expressions of the form $\text{tr}[\,\mathbb{B}_m(\boldsymbol{b})\rho_n]$ where m and n range on $\{0,1\}$. Looking specifically to the special case $\boldsymbol{a} = \boldsymbol{b}$ (Bob and Alice measure spin-squared in identical directions), we find

$$\begin{aligned} &\text{tr}[\,\mathbb{B}_0(\boldsymbol{a})\rho_0] = 1 \qquad && \text{tr}[\,\mathbb{B}_1(\boldsymbol{a})\rho_0] = 0 \\ &\text{tr}[\,\mathbb{B}_1(\boldsymbol{a})\rho_0] = 0 \qquad && \text{tr}[\,\mathbb{B}_1(\boldsymbol{a})\rho_1] = 1 \end{aligned}$$

In short: if the composite system is in the singlet state, and if Alice and Bob use identically aligned spin-squared meters, then they are certain to get identical results (0s with probability $\frac{1}{3}$, 1s with probability $\frac{2}{3}$). This is what Conway and Kochen mean when they say the particles are "twinned."

The situation here is precisely analogous that which arose when Alice and Bob used aligned meters to measure the spin (not spin-squared) of paired spin-one-half particles in the singlet state—the set-up used by Bohm/Bell to demonstrate that "spooky action-at-a-distance" is a fact of nature and that quantum systems in entangled states can exhibit counter-intuitive properties, the set-up that when examined with a *trio* of suitably *non*-aligned spin-meters was shown by Bell to lead from orthodox quantum mechanics to predicted (and experimentally confirmed) results that lie beyond the reach of any plausible non-contextual hidden variable theory. When regarded in this light, the fact that spin-one particles in the singlet state are "twinned" seems not particularly surprising/remarkable.

Statement and proof of the theorem. Alice is equipped with a control panel with 16 buttons, each of which activates one or another of her 16 sets of triplex spin-squared meters, the relative orientations of which conform to Peres' 16-fold set of triads. Alice, her spin-one particle, her experimental equipment and procedures are confined within a spacetime bubble (neighborhood) $\mathcal{A}$. Bob is similarly equipped, confined within a bubble $\mathcal{B}$.

We assume all points of $\mathcal{B}$ to be spacelike separated from all points of $\mathcal{A}$; *i.e.*, that $\mathcal{B}$ is outside the lightcone that extends back/forward from $\mathcal{A}$, and *vice versa*. It follows that the relationship between events in $\mathcal{A}$ and events in $\mathcal{B}$ is necessarily *acausal* because (as Conway & Kochen emphasize) one cannot assign frame-independent meaning to statements concerning which happened first.

Alice elects of her own free will to press one or another of her buttons; she elects, in other words, to measure (in some arbitrarily selected order order) the spin-squared of her particle in the orthogonal triad of directions

$$\{\boldsymbol{a}_i, \boldsymbol{b}_i, \boldsymbol{c}_i\} \quad : \quad i \text{ selected from } \{1, 2, 3, \ldots, 16\}$$

She writes something like

$$\boldsymbol{\theta}_i = \begin{pmatrix} \theta_{ai} \\ \theta_{bi} \\ \theta_{ci} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \text{ else } \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \text{ else } \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

to record the results of her measurements. The phrase "of her own free will" is interpreted here to signify that her decision was not dictated/pre-determined by any event or combination of events in her backward lightcone.

Bob was similarly equipped, but broke up his 3-meter sets so as to produce a set of 33 meters that are collectively capable of measuring spin-squared in each of Peres' 33 directions. He writes

$$\theta_0(\boldsymbol{x}_j) = 0 \text{ else } 1 \quad : \quad j = 1, 2, 3, \ldots, 33$$

to record the results of a measurement.

Because Alice/Bob's respective particles are "twinned" it must be the case that[4]

$$\boldsymbol{\theta}_i = \begin{pmatrix} \theta_0(\boldsymbol{a}_i) \\ \theta_0(\boldsymbol{b}_i) \\ \theta_0(\boldsymbol{c}_i) \end{pmatrix} \quad : \quad \text{invariably (all } i) \tag{3}$$

Alice and Bob are exterior to each other's light cones, acausally related, so it is not possible for Alice's button selection to have been signalled to Bob: his own $\boldsymbol{x}$-selection is necessarily unbiased, is (we assume) freely willed, not dictated/pre-determined by any event or combination of events in his own backward lightcone, but $\frac{3}{16}$ or 18.75% of the time we can expect Bob's $\boldsymbol{x}$ to coincide with one of Alice's chosen $\{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}$ vectors. It is to such cases that (3) pertains.[5] But any $\theta_0(\boldsymbol{x})$ that in all cases conforms to (3) <u>stands necessarily in</u>

[4] Note that two of the elements of the following vector are "counterfactual," in that they refer to the results of measurements that *were not performed* (though they might have been).

[5] The preceding remarks were phrases as though "Alice then Bob" were the sequence of their respective actions, but some intertial observers would reverse the sequence, and some (a minority) would see them to have acted simultaneously.

violation of the Kochen-Specker no-go theorem. We conclude that, unless we are prepared to modify the rudimentary quantum theory and relativity upon which the Conway-Kochen argument hinges, we must abandon our contextual hidden variable hypothesis.

Purported implications. Conway & Kochen attempt to argue that loss of the conjectured contextual hidden variables that were intended to be the "agents of determinism" means that the responses of Alice/Bob's particles are "indeterminate—not dictated/pre-determined by any event or combination of events in their respective backward lightcones"—just as were the decisions Alice/Bob made when selecting from among their experimental options. Here their argument eludes me, for the event that gave birth to the particles (and placed them in the entangled singlet state) certainly lies in the intersection of their backward lightcones, and is, according to orthodox quantum theory, responsible for the way they respond to spin-squared measurements.

But if we accept Conway/Kochen's characterization of what they have accomplished—and accept as well their tacit assumption that what holds for twinned spin-one particles holds universally, for quantum-systems-in-general—then we understand the force of their claim that "*to the extent that we can attribute 'free will' to the decisions Alice/Bob made during the execution of their measurements, so also must we attribute 'free will' to the response of those particles to measurement*." Unembarrassed by their use of anthropocentric language (and protected by their insistence that they do not claim particles to be sentient!) Conway/Kochen go on to assert that particles—since unconstrained by determinism—"do not know how they will respond to measurement until they have done so."

Does that remark add to one's understanding of (say) the quantum mechanical two-slit experiment? Hardly! But it is at least consonant with J. A. Wheeler's paraphrase of Bohr's position, which was that "no [quantum] phenomenon is a phenomenon until it is a recorded (observed) phenomenon." In this regard, Conway & Kochen have come freshly upon an insight that can be traced to near the beginnings of quantum mechancis: that *quantum events truly are as indeterministic as they seem*—precisely the assertion to which Einstein took such strenuous objection.

Commentary. Philosophers have been arguing for centuries about the meaning of "causality," and have taken occasional exception to the uses physicists make of that notion: "output does not precede input," "information transfer is confined to the interior of the lightcone." Physicists themselves have been forced to venture into philosophical deep water when considering the paradoxial physics of tachyons, writing papers with titles like "The tachyonic antitelephone."[6] The closely interrelated concepts of "freedom," "free will,"

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[6] G. A. Benford, D. L. Book & W. A. Newcomb, Phys. Rev. D **2**, 263 (1970). The subject raises possible complications that Conway & Kochen have been content to set aside.

"determinism" have also contributed to the generation of a vast, contentiously inconclusive literature. Some of Conway/Kochen's philosophical colleagues at Princeton have reportedly expressed dismay at the "casual. . . uninformed" use they have made of the free will concept. Other philosophers, in a tradition stretching back through Hume to Hobbs, have been at pains to promote the "compatibilist" view that free will is not inconsistent with strict determinism—contrary to what Conway & Kochen tactily presume.[7]

Gerard 't Hooft has, for about a decade, been exploring the possibility that quantum mechanics is emergent from a fully deterministic physics operative at the Planck scale.[8] 't Hooft accepts Conway/Kochen's argument, but denies their conclusion, since he considers himself forced by his determinism to deny that Alice/Bob have free will.

The Free Will Theorem has been criticised on more technical grounds by Roderich Tumulka[9] and by Sheldon Goldstein *et al.*[10] The latter purport to demonstrate, for example, that "for stochastic models [which Conway/Kochen summarily dismiss from consideration] their conclusion is not correct, while for deterministic models it is not new."

The EPR/Bohm argumentexperiment proceeds from the assumption that spin-one-half particles have been placed in a special entangled state (the singlet state). Bell makes a similar assumption *en route* to demonstrated quantum violation of the Bell inequality (demonstrated impossibility of non-contextual hidden variables). It is interesting that the Kochen-Specker argument arrives at the same conclusion without the presumption of state specialization. On the other hand, to demolish the contextual hidden variable hypothesis Conway & Kochen once again do have to place their particles in a specialized (singlet) state. Very recently, G. Kirchmair *et al*[11] have provided state-independent experimental confirmation of the correctness of the Kochen-Specker proof that non-contextual hidden variables cannot account for the quantum mechanical facts. Perhaps it will not be long before someone reports state-independent experimental evidence that the contextual hidden variable hypothesis is untenable.

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[7] For a philosopher's careful discussion of the Free Will Theorem, visit http://philosophy.ucsd.edu/faculty/wuthrich/PhilPhys/MenonTarun2009Man_FreeeWillThm.pdf.

[8] See "How does God play dice: (Pre)Determinism at the Planck scale," arXiv:hep-th/0104219v1 25 Apr 2001 or "Determinism beneath quantum mechanics," arXiv:quant-ph/0212095v1 16 Dec 2002.

[9] "Comment on 'The Free Will Theorem,' " http://www.maphy.uni-tuebingen.de/members/rotu/papers/freewilly.pdf.

[10] "What does the Free Will Theorem actually prove?" http://math.rutgers.edu/∼oldstein/papers/fwtGTTZ.pdf.

[11] "State-independent experimental test of quantum contextuality," arXiv:0904.1655v2 [quant-ph] 5 May 2009. The experiment exploited recent developments in "non-demolition" technology.