**GEOMETRIC OPTICS**

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*Introduction*

01° We have two objectives: (1) to explain the rainbow, and (2) to describe the relation between light and color. To these ends, we present a simple model of light. The range of validity for the model is limited, but sufficient.

*Photons*

02° We imagine that light is composed of massless, chargeless particles, called photons, the nature of which is enigmatic in the extreme. Each photon carries a characteristic energy, which figures in the interaction between the photon and matter. We represent a photon by the symbol $\phi$ and the corresponding energy by $\epsilon$, more precisely $\epsilon_\phi$.

03° In a vacuum or in a given medium, such as air, water, glass, or diamond, photons travel in straight line paths at constant speeds. The speed of a photon depends, in general, upon the given medium and upon the corresponding energy. We represent the speed of the photon $\phi$ by $\sigma$, more precisely $\sigma_\phi$. When confronted with several media, we refine the notation accordingly.

04° In a vacuum, all photons travel at the same speed, the universal speed of light, $c$:

$$c = 2.998 \cdot 10^8 \, \frac{meters}{second}$$

The same is (very nearly) true in air. In general, the speeds of all photons $\phi$ (no matter the medium) are bounded above by $c$ : $\sigma_\phi \leq c$. Respecting common practice, we introduce the ratio:

$$n_\phi = \frac{c}{\sigma_\phi}$$

called the index of refraction for the photon $\phi$ (in the relevant medium).

*Reflection and Refraction*

05° Encountering an interface between two media, a photon $\phi$ suffers either *reflection* or *refraction*. In the former case, it rebounds from the interface, remaining in the first medium. In the latter case, it enters the second medium. In both cases, the photon travels in straight line paths before and after it meets the interface. The three paths all lie in a common plane, together with the straight line which stands perpendicular to the interface at the point of encounter. The energy of the photon remains the same.

06° There is a third possibility, that the photon yields its energy to the second medium, and disappears. For instance, a photon may contribute its energy (adequate but not destructive) to a photoreceptor in the retina of the eye, thus may initiate a visual response. The range of energies for such "visible" photons is (roughly) the following:

$$\epsilon' \leq \epsilon \leq \epsilon''$$

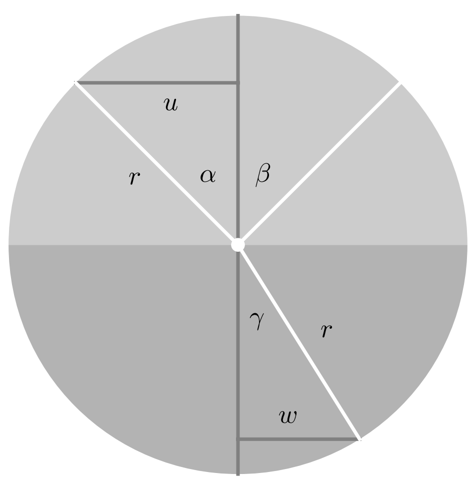
where:

$$\epsilon' = 300\ zjoules\ , \quad \epsilon'' = 500\ zjoules$$

See articles 22°, 23°, and 24°.

07° The geometries of reflection and refraction are governed by the Law of Heron and the Law of Snell. These geometries are illustrated in the following figure and the laws themselves are expressed in the labels to the figure. The white lines represent the straight line paths followed by $\phi$. The symbols $n'_\phi$ and $n''_\phi$ stand for the indices of refraction for $\phi$ in the two media.

**Heron:** $\alpha = \beta$




**Snell:** $n''_\phi\, sin(\gamma) = n'_\phi\, sin(\alpha)$

08° The angles $\alpha$, $\beta$, and $\gamma$ are the incidence, the reflection, and the refraction angles. From trigonometry, we have:

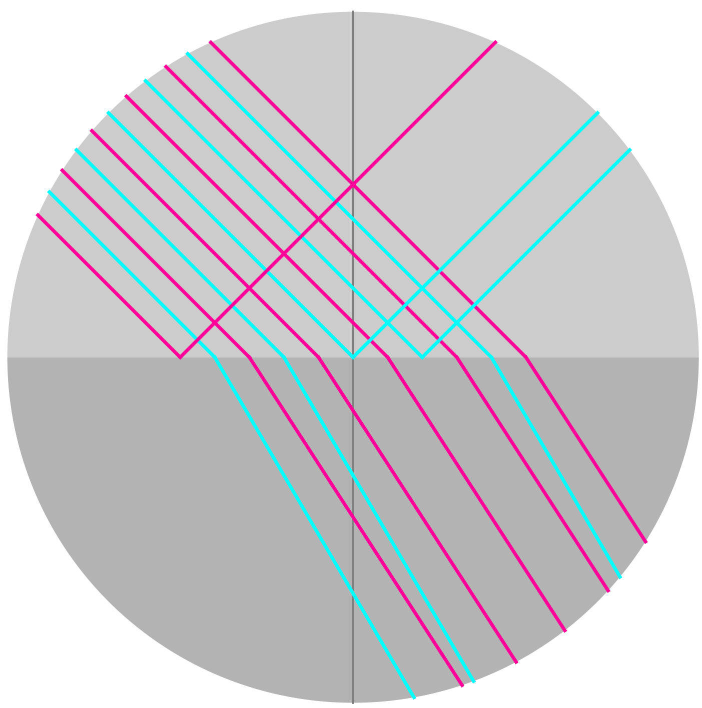
$$sin(\alpha) = \frac{u}{r}, \quad sin(\gamma) = \frac{w}{r}$$

*Beams*

09° In a given medium, let us imagine a stream of photons traveling along mutually parallel straight line paths. The corresponding energies may vary from photon to photon. The number of photons may be legion. We refer to such a stream as a beam of light. Of course, we may refer to the direction of the beam, without ambiguity.

*Reflection and Refraction Redux*

10° In the following diagram, we suggest the spectacle of reflection and refraction for a beam of light, at the interface between a vacuum (or air) and (let us say) water.



**Dispersion**

*Intensity*

11° Now we imagine a beam $B$ of (visible) light in a vacuum, or in air. We want to describe the intensity (that is, the energy per unit time) of the beam, in useful terms.

12° To that end, we introduce a plane perpendicular to the direction of the beam. We introduce lower and upper bounds $\epsilon'$ and $\epsilon''$ for the energies of the various photons which compose the beam. We introduce a partition of the interval formed by the bounds, by marking very closely spaced energies between the bounds:

$$\epsilon' = \epsilon_0 < \epsilon_1 < \epsilon_2 < \epsilon_3 < \cdots < \epsilon_j < \cdots < \epsilon_n = \epsilon''$$

For any time $t$ and for any index $j$ $(0 \leq j < n)$, we observe the photons $\phi$ which pass through the plane in one second, but just those for which:

$$\epsilon_j \leq \epsilon_\phi \leq \epsilon_{j+1}$$

We measure the total energy of the photons so observed. In this way, we obtain (at least in principle) the intensity $I$ of the beam at time $t$, graded by energy.

13° The measurements just described may prove to be independent of time. In such cases, we say that the beam is uniform.

14° Of course, we must rely on the experimentalists to find practical ways to make the foregoing (conceptual) measurements.

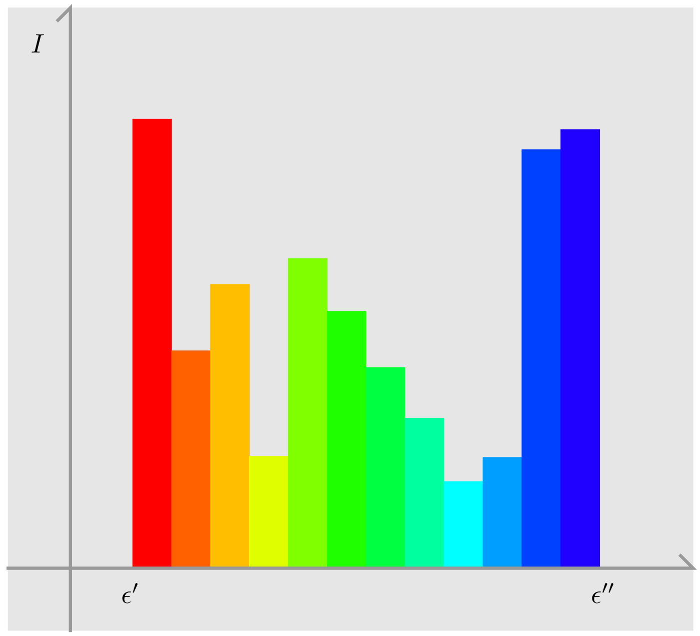
15° In the following figure, we sketch the relation between a uniform beam of visible light and its intensity, in a histogram.

*Outer Lights and Inner Lights*

16° We shall speak of Outer Lights and Inner Lights. The former are elements of the Physical World. The latter are elements of the Mental World of perception. We suggest uniform beams of visible light (in a vacuum or in air) as models for Outer Lights.

17° For Inner Lights, we will follow Erwin Schrödinger (1920) and others to describe a three dimensional space of colors, Color Space. The colors are distinguished by hue, saturation, and brightness. The representation of perceived color is the first step toward understanding the relation between Outer Lights and Inner Lights. See the essay:

ChromaticityCoordinates.pdf




**Histogram**

*Discussion Topics*

18• Imagine a beam of light falling upon a glass window pane. The beam enters the pane, then exits on the other side. What is the relation between the directions of the initial beam and the final beam?

19• A corner cube mirror consists of three square faces which form the corner of a cube. The inner surfaces of the faces are mirrors. Make a perspective drawing of it. A beam of light falling upon a corner cube mirror will "reverse direction." Why?

20• In context of reflection and refraction, there is precisely one straight line path of incidence for which the straight line paths of reflection and refraction are perpendicular:

$$\bar{\alpha} \quad \longrightarrow \quad \bar{\beta} + \bar{\gamma} = \frac{\pi}{2}$$

Explain. The corresponding angle $\bar{\alpha}$ of incidence is the Angle of Brewster.

21• Different uniform beams of visible light may determine the same histogram. Show that it is so, by example.

*Joules*

22° Classical Mechanics is based upon the concepts of time, length, mass, force, and energy. The corresponding units of measurement are the second, the meter, the kilogram, the newton, and the joule. By definition, a force of one newton will compel a mass of one kilogram to accelerate at a rate of one meter per second per second, this at a given moment in time. By definition, if a force of one newton be applied to a mass of one kilogram, in the direction of its (necessarily straight line) motion through a distance of one meter, the force will cause the kinetic energy of the mass to increase by one joule.

23° Kinetic energy is the simplest form of energy in Classical Mechanics. It is the "energy of motion." A mass of $m$ kilograms, moving at a speed of $v$ meters per second, has a kinetic energy of:

$$\frac{1}{2}mv^2$$

joules. A joule of energy is a moderate and familiar entity. For instance, a fast ball thrown by Bob Feller at 98 miles per hour would carry approximately 207.5 joules of kinetic energy.

24° By contrast, the massless, chargeless visible photons are wisps. Their energies are measured by an exquisitely small fraction of a joule, the *zeptojoule* (*zjoule*):

$$zjoule = 10^{-21}\, joules$$

The energies carried by visible photons run from 300 to 500 *zeptojoules*. Nevertheless, though they be subtle they be many.

*The Other Model*

25° Let us introduce two more terms, frequency $\nu$ and wave length $\lambda$, and a new universal constant $h$, called Planck's constant, as a companion to the old universal constant $c$:

$$h = 6.626 \cdot 10^{-34}\, joules \cdot seconds$$

By these terms, we may make (superficial) contact with ideas that stem from the other model of light, the wave model.

26° Imagine a photon $\phi$ carrying energy $\epsilon$. We declare that the frequency of $\phi$ is $\nu$, where:

$$\epsilon = h\nu$$

Imagine a medium (such as a vacuum or air, glass, or water). Let $\sigma$ be the speed of $\phi$ in the medium. (Recall that in a vacuum, $\sigma = c$.) We declare that the wave length of $\phi$ in the medium is $\lambda$, where:

$$\sigma = \nu\lambda$$

In brief, $\epsilon$ and $\nu$ are intrinsic properties of the photon $\phi$ (which do not change), while $\sigma$ and $\lambda$ are variable properties of $\phi$ which depend on the medium.

27° In article 1°, one finds the apology: "The range of validity for the model is limited, but sufficient." But we may ask whether the model (sufficient or not) makes counter intuitive statements. Does it seem plausible that, in a given medium, the larger the energy of the photon the smaller its speed? Yet it must be so, at least for water, because blue lies below red in the (primary) rainbow. The following references comment on the matter:

(1) Francis Sears, Optics (third edition)
(2) Grant Fowles, Modern Optics (second edition)
(3) Eugene Hecht, Optics (fourth edition)

28° "The present standpoint of physicists, in the face of apparently contradictory experiments, is to accept the fact that light appears to be dualistic in nature. The phenomenon of light propagation may best be explained by the electromagnetic wave theory, while the interaction of light with matter, in the processes of emission and absorption, is a corpuscular phenomenon." (Sears, page 3)

29° "While the speed of light waves in a vacuum is the same for all wave lengths, the speed in a material substance depends on the wave length, and hence the index of refraction of a substance is a function of wave length also. The index of refraction is in all cases larger for the shorter wave lengths." (Sears, page 49)

30° "One of the fundamental relations applying to any sort of wave motion is that the speed of propagation $\sigma$ equals the product of the wave length $\lambda$ and the frequency $\nu$: $\sigma = \nu\lambda$. The speed of propagation of light in a transparent medium is not (in general) equal to the speed of light in a vacuum. When light passes from a vacuum into a transparent medium, or from one medium into another having a different index of refraction, the frequency is the same in both media. Hence the wave length changes whenever the speed changes." (Sears, page 19)

31° "Actually, the index of refraction is found to vary with the frequency of the radiation. This is true for all transparent optical media. The variation of the index of refraction with frequency is called dispersion. The dispersion of glass is responsible for the familiar splitting of light into its component colors by a prism." (Fowles, page 7)

32° "The processes of transmission, reflection, and refraction are macroscopic manifestations of scattering on a submicroscopic level." (Hecht, page 83)