MATHEMATICS 391
ASSIGNMENT 08
Due: November 04, 2015

01$^\bullet$ Let $(X, \mathcal{A}, \pi)$ be a probability space. Let F be a random variable and $\mu = F_*(\pi)$ be the corresponding distribution. Specifically, let μ be determined as follows:

$$\mu(\{0\}) = 0.2, \;\; \mu(\{1\}) = 0.8$$

Compute the mean and the standard deviation for F:

$$m = 0.8, \;\; s = 0.4$$

Let:

$$F_1, F_2, F_3, \; \dots$$

be an independent sequence of random variables having common distribution μ. Let $n = 10000$. Let A be the set in $\mathcal{A}$ defined by the following condition:

$$x \in A \;\text{ iff }\; 7920 \leq (F_1(x) + F_2(x) + \;\cdot\; + F_n(x)) \leq 8080$$

Apply the Central Limit Theorem to estimate $\mu(A)$. You should obtain:

$$\frac{1}{\sqrt{2\pi}} \int_{-2}^{2} exp(-\frac{1}{2}y^2)dy$$

02$^\bullet$ Let $(X, \mathcal{A}, \pi)$ be a probability space. Let F be a random variable and $\mu = F_*(\pi)$ be the corresponding distribution. Specifically, let λ be a positive number and let μ be determined as follows:

$$\mu(\{n\}) = exp(-\lambda)\frac{1}{n!}\lambda^n \qquad (n = 0, 1, 2, \; \dots)$$

Verify (again) that both the mean and the variance of μ equal λ. Let:

$$F_1, F_2, F_3, \; \dots$$

be an independent sequence of random variables having common distribution μ. Let n be a positive integer. Let a and b be nonnegative numbers for which $a < b$. Let A be the set in $\mathcal{A}$ defined by the following condition:

$$x \in A \;\text{ iff }\; a \leq (F_1(x) + F_2(x) + \;\cdot\; + F_n(x)) \leq b$$

Apply the Central Limit Theorem to estimate $\mu(A)$. You should obtain a number of the form:

$$\frac{1}{\sqrt{2\pi}} \int_{r}^{s} exp(-\frac{1}{2}y^2)dy$$

where r and s are numbers for which $r < s$

03• Review the definition of the distribution μ in the foregoing problem. One refers to it as the Poisson Distribution, with parameter λ. Calculate the Characteristic Function for μ:

$$\hat{\mu}(t) = \int_{\mathbf{R}} exp(ity)\mu(dy) = exp(\lambda(exp(it) - 1))$$

where t is any real number.

04• Apply your developing understanding of Characteristic Functions to show that, for any two independent random variables F and G, if the distribution for F is Poisson with parameter α and if the distribution for G is Poisson with parameter β then the distribution for $F + G$ is Poisson with parameter $\alpha + \beta$.