MATHEMATICS 391
ASSIGNMENT 6
Due: October 14, 2015

01° Let $\mathbf{S}^2$ be the unit sphere in $\mathbf{R}^3$, consisting of all points P for which:

$$\|P\| = \sqrt{P \bullet P} = 1$$

Let $\mathcal{A}$ be the σ-algebra composed of all "reasonable" (that is, *borel*) subsets of $\mathbf{S}^2$. Let μ be the surface area measure on $\mathcal{A}$, normalized by division by 4π. Consequently, $(\mathbf{S}^2, \mathcal{A}, \mu)$ is a probability space. Let N be the north pole in $\mathbf{S}^2$:

$$N = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Let F be the random variable defined on $\mathbf{S}^2$ which assigns to each point P the distance between P and N:

$$F(P) = \|P - N\|$$

Calculate the first and second moments of F, hence the variance of F.

02° In context of the foregoing probability space $(\mathbf{S}^2, \mathcal{A}, \mu)$, let Φ and Θ be the random variables defined by assigning to each point P in $\mathbf{S}^2$ the *longitude* $\phi = \Phi(P)$ and the *latitude* $\theta = \Theta(P)$ of P:

$$P = (cos(\theta)cos(\phi),\ cos(\theta)sin(\phi),\ sin(\theta))$$

Calculate the marginal distributions for Φ and Θ. Then determine whether or not Φ and Θ are independent.