MATHEMATICS 391
ASSIGNMENT 2
Due: September 16, 2015

$01^\bullet$ **Elementary Probability**: article $38^\bullet$.

$02^\bullet$ Let X be a (nonempty) finite set and let P be a probability function on $\mathcal{P}(X)$. Let Y be a finite set of real numbers. Let F be a mapping carrying X to Y. One defines the probability function Q on $\mathcal{P}(Y)$ as follows:

$$Q(B) := P(F^{-1}(B))$$

where B is any subset of Y. One refers to Q as the *distribution* of F (relative, of course, to P). For each positive integer k, one defines the *k-th moment* $m_k(F)$ of F as follows:

$$m_k(F) := \sum_{x\in X} F(x)^k P(x) = \sum_{y\in Y} y^k Q(y)$$

See article $41^\bullet$ in the booklet **Elementary Probability**. One refers to $m(F) \equiv m_1(F)$ as the *mean* of F or, very often, as the *expectation* of F. One defines the *variance* $v(F)$ of F as follows:

$$v(F) = m_2(F) - m_1(F)^2$$

Verify that:

$$v(F) = \sum_{x\in X} (F(x) - m(F))^2 P(x) = \sum_{y\in Y} (y - m(F))^2 Q(y)$$

One defines the *standard deviation* $s(F)$ of F as the square root of the variance:

$$s(F) = \sqrt{v(F)}$$

Now let n be a positive integer. With reference to articles 34°, 36°, 37°, and (especially) $43^\bullet$ in the booklet **Elementary Probability**, find the mean and the variance of the random variable $\bar{F}$ defined on the bernoulli n-trial space $(\bar{X}, \bar{P})$ by assigning to each trial:

$$\bar{x} = \epsilon_1\epsilon_2 \ \cdots \ \epsilon_n$$

the number of "successes" which occur in it:

$$\bar{F}(\bar{x}) = \sum_{j=1}^{n} \epsilon_j$$

03• With reference to article 53° in the booklet **Elementary Probability**, prove that, for any y in Y and z in Z:

$$Q_z(y) = \frac{R_y(z)Q(y)}{\sum_{\bar{y}\in Y} R_{\bar{y}}(z)Q(\bar{y})}$$

and:

$$R_y(z) = \frac{Q_z(y)R(z)}{\sum_{\bar{z}\in Z} Q_{\bar{z}}(y)R(\bar{z})}$$

These relations are the formal statements of **Bayes' Theorem**. Apply the theorem to the following context. ......