MATHEMATICS 331
ASSIGNMENT
Due: April 2, 2015

01° Let **V** be a finite dimensional linear space. Let $\mathbf{N}_1$ and $\mathbf{N}_2$ be nilpotent linear mappings carrying **V** to itself, for which:

$$N_1 N_2 = N_2 N_1$$

Show that $N_1 + N_2$ is nilpotent. To that end, introduce positive integers k and ℓ for which:

$$N_1^k = 0, \quad N_2^\ell = 0$$

Then prove that:

$$(N_1 + N_2)^{k+\ell} = 0$$

You will benefit by reviewing Professor Thomas Moriarty's monograph on the Binomial Theorem.

02° Let **V** be a finite dimensional linear space, for which the scalar field is **C**. Let the dimension of **V** be n. Let L be a linear mapping carrying **V** to itself. Let ϕ be the characteristic polynomial for L:

$$\phi(\zeta) = det(\zeta I - L)$$

where ζ is any number in **C**. Of course, we may present ϕ as follows:

$$\phi(\zeta) = \prod_{j=1}^{r} (\zeta - a_j)^{d_j}$$

where:

$$a_1, a_2, \ \ldots \ , a_r$$

are the characteristic values, mutually distinct. Show that:

$$det(L) = (-1)^n \prod_{j=1}^{r} a_j^{d_j}$$

03° Find polynomials f and g such that:

$$f(z)(z^3 - 1) + g(z)(z^3 + 1) = 1$$

where ζ is any complex number, or show that it cannot be done.

04° Let M be a linear mapping carrying $\mathbf{R}^6$ to itself, for which the square array stands as follows:

$$\bar{M} = \begin{pmatrix} 6 & 1 & 0 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & 1 & 0 & 0 \\ 0 & 0 & 0 & 6 & 1 & 0 \\ 0 & 0 & 0 & 0 & 6 & 1 \\ 0 & 0 & 0 & 0 & 0 & 6 \end{pmatrix}$$

Find the smallest positive integer ν for which:

$$(M - 6I)^{\nu} = 0$$