MATHEMATICS 331
ASSIGNMENT 4
Due: February 19, 2015

01° Let **P** be the set of all polynomial functions of the form:

$$P(x) = \sum_{j=0}^{5} c_j x^j$$

where c_0, c_1, c_2, c_3, c_4, and c_5 are any (real) numbers. Let **P** be supplied with the familiar operations of addition and scalar multiplication. Note that the following sequence $\mathcal{P}$ of six members of **P** is a basis for **P**:

$$\mathcal{P}: \quad \begin{aligned} P_0(x) &= x^0 = 1 \\ P_1(x) &= x^1 = x \\ P_2(x) &= x^2 \\ P_3(x) &= x^3 \\ P_4(x) &= x^4 \\ P_5(x) &= x^5 \end{aligned}$$

Let L be the mapping carrying **P** to **R**, defined as follows:

$$L(P) = P^{\circ}(1) - \int_0^2 P^{\circ\circ\circ}(y)dy$$

where P is any member of **P**. Verify that L is a linear functional. Find the matrix Λ for L relative to the basis $\mathcal{P}$ for **P** and the standard basis $\mathcal{E}$ for **R**. Describe the rectangular array M corresponding to Λ.

02° Let **V** be a finite dimensional linear space, having dimension 6. Let $\mathcal{B}$ be a basis fir **V**:

$$\mathcal{B}: \quad B_1, B_2, B_3, B_4, B_5, B_6$$

Let $\mathbf{V}^*$ be the linear space which consists of all linear functionals defined on **V**. Such functionals are, by definition, linear mappings Z carrying **V** to **R**. Let $\mathcal{Z}$ be the basis of $\mathbf{V}^*$ corresponding to the basis $\mathcal{B}$ for **V**:

$$\mathcal{Z}: \quad Z_1, Z_2, Z_3, Z_4, Z_5, Z_6$$

The following relation between $\mathcal{B}$ and $\mathcal{Z}$ defines and characterizes $\mathcal{Z}$:

$$Z_j(B_k) = \begin{cases} 0 & \text{if } j \neq k \\ 1 & \text{if } j = k \end{cases}$$

Now let L be a linear mapping carrying $\mathbf{V}$ to itself, let Λ be the matrix defined by L relative to the bases $\mathcal{B}$ and $\mathcal{B}$ for $\mathbf{V}$ and $\mathbf{V}$, and let M be the corresponding rectangular array, having 6 rows and 6 columns:

$$\begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} & m_{15} & m_{16} \\ m_{21} & m_{22} & m_{23} & m_{24} & m_{25} & m_{26} \\ m_{31} & m_{32} & m_{33} & m_{34} & m_{35} & m_{36} \\ m_{41} & m_{42} & m_{43} & m_{44} & m_{45} & m_{46} \\ m_{51} & m_{52} & m_{53} & m_{54} & m_{55} & m_{56} \\ m_{61} & m_{62} & m_{63} & m_{64} & m_{65} & m_{66} \end{pmatrix}$$

Show that:

$$m_{jk} = Z_j(L(B_k))$$

where $1 \leq j \leq 6$ and $1 \leq k \leq 6$.

03° Let M be a rectangular array having 2 rows and 2 columns:

$$M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

One defines the *determinant* of M as follows:

$$det(M) = m_{11}m_{22} - m_{21}m_{12}$$

Let P be the quadratic polynomial defined as follows:

$$P(x) = det(xI - M)$$

where x is any number and where, of course, I is identity array having 2 rows and 2 columns:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Find the coefficients a, b, and c for P:

$$P(x) = ax^2 + bx + c$$

Show that:

$$aM^2 + bM + cI = 0$$