MATHEMATICS 331
ASSIGNMENT 1
Due: January 29, 2015

01° Let **V** be the set of all polynomial functions of the form:

$$p(x) = a + bx + cx^2 + dx^3$$

where a, b, c, and d are any real numbers. Of course, one should interpret x as a real variable. Let L be the mapping carrying **V** to itself, defined as follows:

$$L(p) = p^\circ$$

where p is any polynomial function in **V**. By p°, we mean the derivative of p with respect to x. Under the familiar operations of addition and scalar multiplication:

$$(p_1 + p_2)(x) = (a_1 + a_2) + (b_1 + b_2)x + (c_1 + c_2)x^2 + (d_1 + d_2)x^3$$
$$(u.p)(x) = ua + ubx + ucx^2 + udx^3$$

V is a linear space. Verify that L is a linear mapping. Describe the kernel and the range of L:

$$ker(L), \quad ran(L)$$

02° Let **V** be the set of all twice differentiable real valued functions of the real variable x for which:

$$f^{\circ\circ}(x) + f(x) = 0$$

Under the familiar operations of addition and scalar multiplication, **V** is a linear space. Of course, the trivial function $\bar{0}$ having constant value 0 lies in **V**. Find two nontrivial functions f_1 and f_2 in **V** such that neither is a scalar multiple of the other.