MATHEMATICS 322
ASSIGNMENT 11
Due: December 02, 2015

01• Let J be an open interval in $\mathbf{R}$ containing 0. Let n be a positive integer. Let A be a continuous mapping carrying J to the linear space composed of all matrices (with real entries) having n rows and n columns:

$$A: \quad t \longrightarrow A(t) = \{A_{jk}(t)\}_{j,k=1}^{n}$$

Let Q be any matrix (with real entries) having n rows and n columns:

$$Q = \{Q_{jk}\}_{j,k=1}^{n}$$

Let P be a differentiable mapping carrying J to the linear space composed of all matrices (with real entries) having n rows and n columns:

$$P: \quad t \longrightarrow P(t) = \{P_{jk}(t)\}_{j,k=1}^{n}$$

which provides the solution to the following linear ODE:

$$P'(t) = A(t)P(t), \quad P(0) = Q \qquad (t \in J)$$

Show that:

$$det\, P(t) = det\, Q\, exp(\int_0^t tr\, A(s)ds) \qquad (t \in J)$$