MATHEMATICS 322
ASSIGNMENT 3
Due: September 23, 2015

Fourier Series

1• Let h be a complex valued function defined and continuous on $\mathcal{R}$. Let h be periodic with period 2π. One defines the *even* and *odd* parts of h as follows:

$$f(t) \equiv \frac{1}{2}(h(t) + h(-t)), \ \ g(t) \equiv \frac{1}{2}(h(t) - h(-t)) \qquad (t \in \mathcal{R})$$

Obviously:

$$h(t) = f(t) + g(t), \ \ f(-t) = f(t), \ \ g(-t) = -g(t)$$

The Fourier Coefficients for f, g, and h stand as follows:

$$\begin{aligned} a_j &\equiv \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)e^{-ijt}dt \\ b_j &\equiv \frac{1}{2\pi}\int_{-\pi}^{\pi} g(t)e^{-ijt}dt \qquad (j \in \mathcal{Z}) \\ c_j &\equiv \frac{1}{2\pi}\int_{-\pi}^{\pi} h(t)e^{-ijt}dt \end{aligned}$$

Show that:

$$c_j = a_j + b_j, \ \ a_j = \frac{1}{2}(c_j + c_{-j}), \ \ b_j = \frac{1}{2}(c_j - c_{-j})$$

so that:

$$a_{-j} = a_j, \ \ b_{-j} = -b_j, \ \ a_0 = c_0, \ \ b_0 = 0$$

Let h be twice continuously differentiable. By the Theorem of Fourier:

$$\begin{aligned} f(t) &= \sum_{j=-\infty}^{\infty} a_j e^{ijt} \\ g(t) &= \sum_{j=-\infty}^{\infty} b_j e^{ijt} \\ h(t) &= \sum_{j=-\infty}^{\infty} c_j e^{ijt} \end{aligned}$$

Now let:

$$\alpha_j \equiv 2a_j, \quad \beta_j \equiv 2ib_j$$

Show that:

$$\alpha_j = \frac{2}{\pi}\int_0^{\pi} f(t)cos(jt)dt$$
$$\beta_j = \frac{2}{\pi}\int_0^{\pi} g(t)sin(jt)dt$$

Then verify the Fourier Cosine Series:

$$f(t) = \frac{1}{2}\alpha_0 + \sum_{j=1}^{\infty} \alpha_j cos(jt)$$

and the Fourier Sine Series:

$$g(t) = \sum_{j=1}^{\infty} \beta_j sin(jt)$$

Green

$2^\bullet$ Study Theorem A in Chapter 3 of our Notebook. We will discuss this theorem in the lectures.

Sturm/Liouville

$3^\bullet$ Study Theorem B in Chapter 3 of our Notebook. We will discuss this theorem in the lectures.

$4^\bullet$ ......

5° ......