MATHEMATICS 321
ASSIGNMENT 5
Due: October 7, 2015

01$^\bullet$ Let P_1, P_2, and P_3 be three distinct points in $\mathbf{R}^2$. Let F_1, F_2, and F_3 be the mappings carrying $\mathbf{R}^2$ to itself, defined as follows:

$$F_1(X) = \frac{1}{2}(X + P_1), \quad F_2(X) = \frac{1}{2}(X + P_2), \quad F_3(X) = \frac{1}{2}(X + P_3)$$

where X is any point in $\mathbf{R}^2$. Note that F_1, F_2, and F_3 are contraction mappings, with contraction constants having the common value 1/2. Let $\mathcal{F}$ be the mapping carrying $\mathcal{H}(\mathbf{R}^2)$ to itself, defined as follows:

$$\mathcal{F}(L) = F_1(L) \cup F_2(L) \cup F_3(L)$$

where L is any member of $\mathcal{H}(\mathbf{R}^2$. Show that $\mathcal{F}$ is a contraction mapping. What is the contraction constant for $\mathcal{F}$? Let T be the (closed) triangular area defined by P_1, P_2 and P_3. Draw a picture of the set:

$$K = \mathcal{F}^3(T)$$

in $\mathcal{H}(\mathbf{R}^2)$.

02$^\bullet$ Let $\mathbf{R}$ be the set of all real numbers, supplied with the usual metric. Let f be a continuous complex valued function defined on $\mathbf{R}$. We say that f has *compact support* iff there is a compact subset K of $\mathbf{R}$ such that, for each number x in $\mathbf{R}\backslash K$, $f(x) = 0$. Of course, such a function must be bounded. Let $\mathbf{X}$ be the set of all continuous complex valued functions defined on $\mathbf{R}$, having compact support. Let d be the uniform metric on $\mathbf{X}$:

$$d(f_1, f_2) = \sup_{x \in \mathbf{R}} |f_1(x) - f_2(x)| \qquad (f_1,\ f_2 \in \mathbf{X})$$

Let Q be the mapping carrying $\mathbf{X}$ to itself, defined as follows:

$$Q(f)(x) = xf(x) \qquad (f \in \mathbf{X},\ x \in \mathbf{R})$$

Show that Q is continuous on $\mathbf{X}$ or show that it is not so.

03$^\bullet$ Let X be a metric space. We say that X satisfies Condition $C^\bullet$ iff, for any decreasing sequence:

$$\cdots \subseteq C_j \subseteq \cdots \subseteq C_3 \subseteq C_2 \subseteq C_1$$

of nonempty closed subsets of X, the intersection:

$$\bigcap_{j=1}^{\infty} C_j$$

is nonempty. Show that if X satisfies Condition $C^\bullet$ then X is compact.