MATHEMATICS 321
ASSIGNMENT 4
Due: September 30, 2015

01$^\bullet$ Let A, B, and C be nonempty subsets of $\mathbf{R}^+$ for which:

$$A + B \subseteq C$$

Of course, the foregoing relation means that, for each u and v in $\mathbf{R}^+$, if $u \in A$ and $v \in B$ then $u + v \in C$. Prove that:

$$\inf C \leq \inf A + \inf B$$

02$^\bullet$ Let X_1 and X_2 be metric spaces and let F and G be continuous mappings carrying X_1 to X_2. Let Y be the subset of X_1 consisting of all x in X_1 such that $F(x) = G(x)$. Prove that Y is closed.

03$^\bullet$ Let X be a metric space. Show that if X is totally bounded then X is separable.

04$^\bullet$ Let $I \equiv [\,0, 1\,]$ be the closed unit interval in $\mathbf{R}$. Let $\mathbf{X}$ be the set of all continuous complex valued functions defined on I. Of course, $\mathbf{X}$ is a linear space. Let $\mathbf{X}$ be supplied with the Inner Product:

$$\langle\!\langle\, f, g \,\rangle\!\rangle \equiv \int_I f(t)\overline{g(t)}dt \qquad (f, g \in \mathbf{X})$$

the Integral Norm:

$$\langle\!\langle h \rangle\!\rangle = \sqrt{\langle\!\langle\, h, h \,\rangle\!\rangle} = \sqrt{\int_I |h(t)|^2 dt} \qquad (h \in \mathbf{X})$$

and the Integral Metric:

$$\mathbf{d}(f, g) \equiv \langle\!\langle f - g \rangle\!\rangle = \sqrt{\int_I |f(t) - g(t)|^2 dt} \qquad (f, g \in \mathbf{X})$$

Show that $\mathbf{X}$ is not complete.

05$^\bullet$ Let $\mathbf{S}^2$ be the unit sphere in $\mathbf{R}^3$. Let $\mathbf{E}$ be the subset of $\mathbf{S}^2 \times \mathbf{S}^2 \times \mathbf{S}^2$ consisting of all ordered triples $(\sigma_1, \sigma_2, \sigma_3)$ such that $\sigma_1 \neq \sigma_2$, $\sigma_1 \neq \sigma_3$, and $\sigma_2 \neq \sigma_3$. Let F be the real valued function defined on $\mathbf{E}$ as follows:

$$F(\sigma_1, \sigma_2, \sigma_3) = \frac{1}{\|\sigma_1 - \sigma_2\|} + \frac{1}{\|\sigma_1 - \sigma_3\|} + \frac{1}{\|\sigma_2 - \sigma_3\|}$$

where $(\sigma_1, \sigma_2, \sigma_3) \in \mathbf{E}$. Describe the range of F. Of course, you should prove that your description is correct.