MATHEMATICS 321
ASSIGNMENT 2
Due: September 16, 2015

01$^\bullet$ Let σ be the sequence in the metric space $\bar{\mathbf{M}}((0,1))$ defined as follows:

$$\sigma(j)(t) = t^j \qquad (j \in \mathbf{Z}^+, \ \ 0 < t < 1)$$

Let $\bar{0}$ be the zero function in $\bar{\mathbf{M}}((0,1))$:

$$\bar{0}(t) = 0 \qquad (0 < t < 1)$$

Show that, contrary to appearance, σ does not converge to $\bar{0}$. [In fact, σ does not converge.]

02$^\bullet$ Let $\mathbf{P}((0,1))$ be the subset of $\bar{\mathbf{M}}((0,1))$ consisting of all (complex valued) polynomial functions (restricted, of course, to $(0,1)$). Let D be the mapping carrying $\mathbf{P}((0,1))$ to itself, defined by differentiation:

$$D(p) = p' \qquad (p \in \mathbf{P}((0,1)))$$

Show that D is not continuous at $\bar{0}$. [In fact, D is not continuous at any polynomial in $\mathbf{P}((0,1))$.]

03$^\bullet$ Let $\mathbf{R}^2$ be supplied as usual with the cartesian metric. Describe a subset Y of $\mathbf{R}^2$ such that Y is open but:

$$Y \neq int(clo(Y))$$

04$^\bullet$ Let σ be a sequence in $\mathbf{R}$. Show that there must be a subsequence τ of σ such that τ is *monotone*. We mean to say that τ is increasing:

$$j \in \mathbf{Z}^+ \Longrightarrow \tau(j) \leq \tau(j+1)$$

or that τ is decreasing:

$$j \in \mathbf{Z}^+ \Longrightarrow \tau(j+1) \leq \tau(j)$$