MATHEMATICS 311
ASSIGNMENT 10
Due: April 22, 2015

01° Show that:

$$\Gamma(1) = 1 \quad \text{and} \quad \Gamma(\frac{1}{2}) = \sqrt{\pi}$$

02° Show that:

$$\Gamma(z+1) = z\Gamma(z)$$

where $0 < Re(z)$. To that end, apply Integration by Parts. Apply Mathematical Induction to infer that:

$$\Gamma(n+1) = n!$$

where n is any nonnegative integer.

03° Show that:

$$\Gamma(1-z)\Gamma(z) = \int_0^\infty \frac{t^{-z}}{1+t}dt = \frac{\pi}{sin(\pi z)}$$

where $0 < Re(z) < 1$. Now we may define:

$$\Delta(z) = \frac{\pi}{sin(\pi z)}\frac{1}{\Gamma(1-z)}$$

where $Re(z) < 1$. Note that Δ has simple poles at:

$$\dots \ , -4, \ -3, \ -2, \ -1, \ 0$$

By the foregoing relation, $\Gamma(z)$ and $\Delta(z)$ coincide when $0 < Re(z) < 1$. As a result, we may say that Γ "extends" uniquely to a meromorphic function on **C**.

04° Show that:

$$\frac{\Gamma(w)\Gamma(z)}{\Gamma(w+z)} = \int_0^1 t^{w-1}(1-t)^{z-1}dt$$

where $0 < Re(w)$ and $0 < Re(z)$.