MATHEMATICS 311
ASSIGNMENT 9
Due: April 15, 2015

01° Evaluate:

$$\int_0^{2\pi} \frac{cos(\theta)}{a - cos(\theta)} d\theta \qquad \text{(where } 1 < a\text{)}$$

02° Let u and v be real numbers and let z be a complex number for which:

$$-1 \le u \le 1,\ v \le 0,\ |z| < 1$$

Show that $1 - 2uz + z^2 \neq v$. Using the foregoing relation, justify the following power series expansion:

$$\frac{1}{\sqrt{1 - 2uz + z^2}} = \sum_{k=0}^{\infty} P_k(u) z^k$$

Calculate the following coefficient functions:

$$P_0,\ P_1,\ P_2,\ P_3$$

They are the first few of the Legendre Polynomials.

03° Let Ω be a region in $\mathbf{C}$. Let S be a subset of Ω. We say that S is a set of *uniqueness* iff, for any functions f and g defined and analytic on Ω, if the restrictions of f and g to S are equal (which is to say that, for any z in S, $f(z) = g(z)$) then $f = g$ (which is to say that, for any z in Ω, $f(z) = g(z)$). Show that if S contains an accumulation point (that is, if S contains a point σ such that, for any positive number r, there is a point τ in S such that $\tau \neq \sigma$ and $|\tau - \sigma| < r$) then S is a set of uniqueness. To that end, you need only review arguments which we have developed in the lectures.

04° Let Ω be a region in $\mathbf{C}$. Let S be a set of uniqueness in Ω. Let:

$$\phi: \quad f_1,\ f_2,\ f_3,\ \ldots$$

be a sequence of functions defined and analytic on Ω which is uniformly bounded on compact subsets of Ω. Show that if ϕ converges pointwise on S then it converges uniformly on compact subsets of Ω. One refers to this (remarkable) implication as the Theorem of Vitali. (For the proof, you should bear in mind the technique of "subsubsequences.")