MATHEMATICS 311
ASSIGNMENT 6
Due: March 11, 2015

01° Calculate:

$$\int_{-\infty}^{\infty} \frac{cos(x)}{exp(x) + exp(-x)} dx$$

02° Let a and b be real numbers for which $0 < b < a$. Calculate:

$$\int_0^{\pi} \frac{1}{(a + b\,cos(\theta))^2} d\theta$$

03° Find the Laurent Expansion:

$$\frac{1}{(z-1)(z-2)} = \sum_{k=-\infty}^{\infty} c_k z^k \qquad (2 < |z|)$$

04° For the polynomial:

$$p(z) = z^9 - 8z^2 + 5$$

show that all the zeros lie in the annular region: $\frac{1}{2} < |z| < \frac{3}{2}$, and that two of them lie in the annular region: $\frac{1}{2} < |z| < 1$.

05° Let $\mathbf{\Delta}$ be the unit disk in $\mathbf{C}$ centered at 0. Let K be a compact subset of $\mathbf{\Delta}$. Let f be a function defined and analytic on $\mathbf{\Delta}$ for which $f(\mathbf{\Delta}) \subseteq K$. Prove that f admits precisely one fixed point.

06• Let $\mathbf{\Delta}$ be the unit disk in $\mathbf{C}$ centered at 0. Let f be the function defined on $\mathbf{\Delta}$ as follows:

$$f(z) = \frac{z}{(1-z)^2} \qquad (z \in \mathbf{\Delta})$$

Show that f is injective. Describe $f(\mathbf{\Delta})$.