MATHEMATICS 311
ASSIGNMENT 4
Due: February 25, 2015

01° Let Ω be a region in $\mathbf{C}$ and let f be a complex-valued function defined and analytic on Ω:

$$f(z) = u(x,y) + iv(x,y) \qquad (z = x + iy \in \Omega)$$

Let ρ be the modulus function for f:

$$\rho(x,y) = |f(z)| \qquad (x + iy = z \in \Omega)$$

Show that, for any $z = x+iy$ in Ω, (x,y) is a local minimum for ρ iff $f(z) = 0$.

02° For each of the following functions, find the isolated singularities. Verify that they are all poles. For each such singularity, find the corresponding residue:

$$f(z) = \frac{z^3}{1 - z^5}, \quad g(z) = \frac{z^5}{(1 - z^2)^2}, \quad h(z) = \frac{cos(z)}{1 + z + z^2}$$

03° From the relation:

$$exp(\frac{t}{2}(z - \frac{1}{z})) = \sum_{n=-\infty}^{\infty} J_n(t) z^n$$

show that:

$$J_n(t) = \frac{1}{\pi}\int_0^{\pi} cos(t\,sin(\theta) - n\theta)d\theta, \quad J_{-n}(t) = (-1)^n J_n(t)$$

Of course, $t \in \mathbf{R}$, $z \in \mathbf{C}$, and $n \in \mathbf{Z}$. One refers to J_n as the *Bessel Function* of order n.

04° Let Ω be a region in $\mathbf{C}$ and let w be a complex number in Ω. Let f be a complex-valued function defined and analytic on Ω. Let f have a zero of order n at w:

$$f(w) = 0,\ \ f'(w) = 0,\ \ \ldots\ \ ,\ \ f^{(n-1)}(w) = 0,\ \ f^n(w) \neq 0$$

Show that:

$$res_w(\frac{f'}{f}) = n$$