MATHEMATICS 212
EXAMINATION
Due: Wednesday, May 13, 2015, NOON, Library 306

01• What is the fifth letter of the Greek alphabet?

02• Let S be the surface in $\mathbf{R}^3$ composed of all points (x, y, z) which satisfy the following conditions:

$$x = r\,cos(\phi), \quad y = r\,sin(\phi), \quad z = \phi$$

where:

$$0 \leq r \leq 1, \quad 0 \leq \phi \leq \pi$$

Draw a picture of S. Find the surface area of S.

03• Let V be the region in $\mathbf{R}^3$ consisting of all points (x, y, z) for which:

$$0 \leq z \leq 2 - x, \quad x^2 + 4y^2 \leq 4$$

Draw a picture of V. Find the volume of V.

04• Let F be the vector field defined on $\mathbf{R}^3$ as follows:

$$F(x, y, z) = (x + z^2, y + z^2, x^2 + y^2)$$

Let c be any positive number. Let S be the surface in $\mathbf{R}^3$ consisting of all points (x, y, z) for which:

$$x^2 + y^2 + \frac{z^2}{c^2} = 1, \quad 0 \leq z$$

Calculate the surface integral of $\nabla \times F$ over S:

$$\int\!\!\int_S \nabla \times F$$

05• Let a, b, and c be positive numbers. Let E be the ellipsoid in $\mathbf{R}^3$ composed of all points (x, y, z) which satisfy the following condition:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1$$

Calculate:

$$\int\!\!\int\!\!\int_E \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}\right)^{1/2} dxdydz$$

06• Let V be the region in $\mathbf{R}^3$ consisting of all points (x, y, z) such that:

$$0 \leq x, \quad 0 \leq y, \quad 0 \leq z, \quad x^2 + y^2 \leq 1, \quad x^2 + z^2 \leq 1, \quad y^2 + z^2 \leq 1$$

Show that the volume of V equals:

$$2 \int_0^{\pi/4} \int_0^1 \sqrt{1 - r^2 cos^2(\phi)} r dr d\phi$$

Do not be discouraged if you cannot evaluate the integral.

07• Let C be the circle in $\mathbf{R}^3$ composed of all points (x, y, z) which satisfy the following conditions:

$$x^2 + y^2 = 1 \quad \text{and} \quad z = 0$$

Let A be the region in $\mathbf{R}^3$ complementary to C: $A = \mathbf{R}^3 \backslash C$. Let λ be the 1 form on A defined as follows:

$$\lambda = \frac{1}{(1 - x^2 - y^2)^2 + z^2} \Big(2xzdx + 2yzdy + (1 - x^2 - y^2)dz \Big)$$

Show that:

$$d\lambda = 0$$

Introduce the simple 1 chain H in A, defined as follows:

$$H(t) = (\sqrt{1 + cos(2\pi t)}, 0, sin(2\pi t))$$

where $0 \leq t \leq 1$. Draw a picture to show that the circle C and the range of H are "linked." Then compute:

$$\int_H \lambda$$

Finally, find a 0 form f such that $df = \lambda$ or show that it cannot be done.

08• Let A be the region in $\mathbf{R}^3$ composed of all points (x, y, z) which satisfy the following condition:

$$z^2 - x^2 - y^2 < 1$$

Let μ be the 2 form on $\mathbf{R}^3$ defined as follows:

$$\mu = log(1 + x^2 + y^2 - z^2) \Big[yzdydz + zxdzdx + 2xydxdy \Big]$$

Show that:

$$d\mu = 0$$

Find all the 1 forms λ on $\mathbf{R}^3$ for which:

$$d\lambda = \mu$$

09• Let f be the function defined on $\mathbf{R}^3$ as follows:

$$f(x,y,z) = z + \frac{1}{2}r^2 - \frac{1}{4}r^4$$

where $r^2 = x^2 + y^2$. Let S be the surface in $\mathbf{R}^3$ defined by the conditions:

$$f(x,y,z) = 0, \qquad 0 \leq r \leq 2$$

Draw a picture of S. Find the surface area 2 form for S. Show that the surface area of S equals:

$$2\pi \int_0^2 r\sqrt{r^2(1-r^2)^2+1}\,dr$$