MATHEMATICS 212
ASSIGNMENT 9
Due: April 15, 2015

01° Let f be a function defined on a region Ω in $\mathbf{R}^3$. Let S be the surface in $\mathbf{R}^3$ defined implicitly by f, as follows:

$$f(x,y,z) = 0 \qquad ((x,y,z) \in \Omega)$$

The 2-form:

$$\sigma = *(\frac{1}{\|\nabla f\|} df)$$

is the area 2-form for S. Why? Now compute the surface area of the parabolic surface S defined as follows:

$$z - x^2 - y^2 = 0, \qquad 0 \leq z \leq c^2 \qquad ((x,y,z) \in \mathbf{R}^3)$$

where c is a positive real number. To do so, design an appropriate 2-chain H.

02° With reference to the first problem, find the surface area of the conical surface S defined as follows:

$$z^2 - x^2 - y^2 = 0, \qquad 0 \leq z \leq c \qquad ((x,y,z) \in \mathbf{R}^3)$$

where c is a positive real number. To do so, design an appropriate 2-chain H.

03° Consider the 2-form:

$$\mu = \frac{1}{r^3}(*(rdr))$$

on $\mathbf{R}^3 \backslash \{\mathbf{0}\}$. Show that:

$$d\mu = 0$$

Suppose that there is a 1-form λ on $\mathbf{R}^3 \backslash \{\mathbf{0}\}$ such that:

$$d\lambda = \mu$$

Apply Stokes' Theorem to show that:

$$\int_H \mu = 0$$

where H is the familiar 2-chain which serves to parametrize $\mathbf{S}^2$. But this cannot be so. Why?