MATHEMATICS 212
ASSIGNMENT 7
Due: March 18, 2015

01° Let r be the function defined on $\mathbf{R}^3$ as usual:

$$r(x, y, z) := \sqrt{x^2 + y^2 + z^2}$$

Calculate:

$$\frac{1}{3}d(*(rdr))$$

[Note that $r^2 = x^2 + y^2 + z^2$. Verify that $rdr = xdx + ydy + zdz$.]

02° Verify that, for any differential forms μ and ν on $\mathbf{R}^3$:

$$d(\mu\nu) = (d\mu)\nu + (-1)^k \mu(d\nu)$$

where μ is a k-form $(0 \leq k \leq 3)$.

03° Let k be an integer for which $0 \leq k \leq 3$. Let λ be a k-form on $\mathbf{R}^3$. By definition, $d\lambda$ is a $(k+1)$-form on $\mathbf{R}^3$. Show that:

$$dd\lambda = 0$$

04° Verify that, for each k-form λ on $\mathbf{R}^3$, $**\lambda = \lambda$.

05° For the 1-form:

$$\lambda = p\,dx + q\,dy + r\,dz$$

on $\mathbf{R}^3$, calculate:

$$\mu = *\,d\lambda$$

We may say that $*d\lambda$ is the *curl* of λ.

06° Let k be an integer for which $0 \leq k \leq 3$. Let μ be a k-form on $\mathbf{R}^3$. We define the *coderivative* of μ as follows:

$$\delta\mu = (-1)^{k+1}(*d*)\mu$$

Note that $\delta\mu$ is a $(k-1)$-form on $\mathbf{R}^3$. Show that:

$$\delta\delta\mu = 0$$

07° For the 1-form:

$$\lambda = p\,dx + q\,dy + r\,dz$$

on $\mathbf{R}^3$, calculate:

$$\mu = \delta\lambda$$

We may say that $\delta\lambda$ is the *divergence* of λ.

08° Let k be an integer for which $0 \leq k \leq 3$. Let ν be a k-form on $\mathbf{R}^3$. We define the *laplacian* of ν as follows:

$$\triangle\nu = (\delta + d)^2\nu = (\delta d + d\delta)\nu$$

Note that $\triangle\nu$ is a k-form on $\mathbf{R}^3$. For the 0-form ϕ, verify that:

$$\triangle\phi = (\delta d)\phi = (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2})\phi$$

For the cases in which $1 \leq k \leq 3$, verify that:

$$\triangle(pdx + qdy + rdz) = (\triangle p)dx + (\triangle q)dy + (\triangle r)dz$$

$$\triangle(udydz + vdzdx + wdxdy) = (\triangle u)dydz + (\triangle v)dzdx + (\triangle w)dxdy$$

$$\triangle(hdxdydz) = (\triangle h)dxdydz$$