MATHEMATICS 212
ASSIGNMENT 4
Due: February 25, 2015

01° Let B be the subset of $\mathbf{R}^3$ defined by the relations:

$$0 \leq x,\ 0 \leq y,\ 0 \leq z,\ x^2 + y^2 + z^2 \leq 1$$

Calculate:

$$\iiint_B xyz\, dxdydz$$

02° Let B be the unit ball in $\mathbf{R}^3$, composed of the points $Q = (x, y, z)$ for which $x^2 + y^2 + z^2 \leq 1$. Let $P = (u, v, w)$ be a point in $\mathbf{R}^3$ for which $1 < u^2 + v^2 + w^2$. Find the average of the distances from P to the various points Q in B.

03° Let r be any positive number. Let n be any positive integer. Let $S_n(r)$ be the subset of $\mathbf{R}^n$ defined by the relations:

$$0 \leq x_1,\ 0 \leq x_2,\ \ldots\ , 0 \leq x_n,\ x_1 + x_2 + \cdots + x_n \leq r$$

Show that:

$$vol(S_n(r)) := \int\int \cdots \int_{S_n(r)} dx_1 dx_2 \cdots dx_n = \frac{r^n}{n!}$$

Argue by induction, using Fubini's Theorem.

04° Let K be the unit vector in $\mathbf{R}^3$ defined as follows:

$$K = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Let a be a real number for which $0 \leq a \leq 1$. Let D be the subset of $\mathbf{R}^3$ consisting of all (nonzero) vectors P:

$$P = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

for which:

$$\|P\| \leq 1 \quad \text{and} \quad a \leq (\frac{1}{\|P\|}.P) \bullet K$$

Calculate:

$$vol(D) = \iiint_D dxdydz$$

and:

$$\iiint_D xdxdydz, \quad \iiint_D ydxdydz, \quad \iiint_D zdxdydz$$

05• Let r and s be numbers for which:

$$0 < r < s$$

Let A be the subset of $\mathbf{R}^3$ defined by the relations:

$$0 \leq u \leq r, \quad -\pi \leq v \leq \pi, \quad -\pi \leq w \leq \pi$$

Let H be the mapping defined as follows:

$$H: \quad (u, v, w) \longrightarrow (x, y, z) \qquad ((u, v, w) \in A)$$

where:

$$(x, y, z) = ((s + u\,cos\,v)cos\,w, \; (s + u\,cos\,v)sin\,w, \; u\,sin\,v)$$

Let B be the range of H. What is it? Compute:

$$vol(B) = \iiint_B dxdydz$$

by change of variables:

$$\iiint_A |det\,DH(u, v, w)|dudvdw$$

06• Let a, c, and h be any positive real numbers. Find the volume of the region C which lies above the plane:

$$z = 0$$

below the plane:

$$x + cz = h$$

and inside the right circular cylinder:

$$x^2 + y^2 \leq a^2$$

Draw a diagram of the region.