

**MATHEMATICS 212**

ASSIGNMENT 3

Due: February 18, 2015

01° Evaluate:

$$\int_c^1 \int_0^{(\arcsin y)/y} y \cos(xy) dx dy \quad (0 < c < 1)$$

02° For the simple closed (parametrized) curve  $\gamma$  in  $\mathbf{R}^2$ :

$$\gamma(\theta) = (x(\theta), y(\theta)) := (4 \cos \theta, 3 \sin \theta) \quad (0 \leq \theta \leq 2\pi)$$

calculate:

$$\frac{1}{2} \int_0^{2\pi} \det \begin{pmatrix} x(\theta) & y(\theta) \\ x'(\theta) & y'(\theta) \end{pmatrix} d\theta$$

Also, calculate the area of the region enclosed by  $\gamma$ .

03° Find the volume of the *chalice*  $K$  in  $R^3$  defined by the following inequalities:

$$(x, y, z) \in K \text{ iff } x^2 + y^2 \leq 1 \text{ and } x^2 + y^2 \leq z \leq 1$$

04° Calculate:

$$\int \int_{\mathbf{R}^2} \frac{1}{(1 + x^2 + y^2)^{3/2}} dx dy$$

05° Let  $a$  be any positive real number. Find the area inside the curve:

$$r = a(1 + \cos \theta)$$

and outside the circle:

$$r = a$$

Note:

$$r^2 = x^2 + y^2, \quad x = r \cos \theta, \quad y = r \sin \theta$$

06° For the ball  $B$  in  $\mathbf{R}^3$  of radius  $\rho$ :

$$r^2 := x^2 + y^2 + z^2 \leq \rho^2$$

calculate:

$$\int \int \int_B \left(1 - \frac{r^2}{\rho^2}\right) dx dy dz$$

07° Calculate:

$$\int \int_R y^{-3} \exp\left(\frac{tx}{y}\right) dx dy$$

for the rectangle  $R$ :

$$0 \leq x \leq t, \quad 1 \leq y \leq t$$

where  $t$  is any real number for which  $1 < t$ .