MATHEMATICS 211
ASSIGNMENT 4
Due: October 1, 2014

01° Let L be the linear mapping carrying $\mathbf{R}^3$ to $\mathbf{R}^2$ for which the matrix relative to the standard bases:

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

for $\mathbf{R}^3$ and $\mathbf{R}^2$, respectively, stands as follows:

$$L = \begin{pmatrix} -1 & 12 & 10 \\ 6 & 6 & 18 \end{pmatrix}$$

Find the *nullspace* $\mathcal{N}(L)$ for L, composed of all vectors X:

$$X = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

in $\mathbf{R}^3$ for which:

$$L(X) = \begin{pmatrix} -1 & 12 & 10 \\ 6 & 6 & 18 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Show that, in fact, $\mathcal{N}(L)$ is a line in $\mathbf{R}^3$ passing through the origin. Find the *rangespace* $\mathcal{R}(L)$ for L, composed of all vectors Y:

$$Y = \begin{pmatrix} p \\ q \end{pmatrix}$$

in $\mathbf{R}^2$ for which there exists a vector X:

$$X = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

in $\mathbf{R}^3$ such that:

$$L(X) = \begin{pmatrix} -1 & 12 & 10 \\ 6 & 6 & 18 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix} = Y$$

Show that, in fact, $\mathcal{R}(L) = \mathbf{R}^2$.

02° Let L be the mapping carrying $\mathbf{R}^2$ to $\mathbf{R}^3$, defined as follows:

$$L(\begin{pmatrix} s \\ t \end{pmatrix}) = (s-t)\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + (s+t)\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

where s and t are any real numbers. Note that L a linear mapping. Find the matrix:

$$\begin{pmatrix} * & * \\ * & * \\ * & * \end{pmatrix}$$

which defines L.

03° Calculate the determinant of the following matrix:

$$\begin{pmatrix} -1 & 3 & 2 & 1 \\ 2 & -3 & 1 & -1 \\ 0 & 1 & 2 & 2 \\ 4 & 1 & 1 & -1 \end{pmatrix}$$

To that end, apply the characteristic properties of determinants.

04° Calculate the determinant of the following *rook placement* matrix:

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

05° Let L be the linear mapping carrying $\mathbf{R}^2$ to $\mathbf{R}^2$, defined by the following matrix, having 2 rows and 2 columns:

$$L = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$$

where a, b, c, and d are any real numbers. Let A be the subset of $\mathbf{R}^2$ consisting of all vectors:

$$X = \begin{pmatrix} u \\ v \end{pmatrix}$$

for which $0 \leq u \leq 1$ and $0 \leq v \leq 1$. Let B be the image of A under L, consisting of all vectors:

$$Y = \begin{pmatrix} p \\ q \end{pmatrix}$$

in $\mathbf{R}^2$ for which there is some vector X:

$$X = \begin{pmatrix} u \\ v \end{pmatrix}$$

in A such that:

$$L(X) = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix} = Y$$

Show that the area of B equals:

$$|ad - bc| = |det(L)|$$

06° Let a, b, and c be any numbers. Show that:

$$det \begin{pmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{pmatrix} = (c - b)(c - a)(b - a)$$

07• Let c and d be positive constants. Let E be the subset of $\mathbf{R}^2$ composed of all positions:

$$Z = \begin{pmatrix} x \\ y \end{pmatrix}$$

in $\mathbf{R}^2$ such that:

$$\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = d$$

In terms of c and d, find the positive constants a and b such that, for any position:

$$Z = \begin{pmatrix} x \\ y \end{pmatrix}$$

in $\mathbf{R}^2$, Z lies in E iff:

$$(\frac{x}{a})^2 + (\frac{y}{b})^2 = 1$$

You should express a and b in terms of c and d. One refers to E as an *ellipse* with *focii* at:

$$\begin{pmatrix} -c \\ 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} c \\ 0 \end{pmatrix}$$

Draw a picture of E, displaying the focii and indicating the significance of a and b.