

MATHEMATICS 211
ASSIGNMENT 1
Due: September 10, 2014

01• Memorize the Greek alphabet:

α	alpha	A
β	beta	B
γ	gamma	Γ
δ	delta	Δ
ϵ	epsilon	E
ζ	zeta	Z
η	eta	H
θ	theta	Θ
ι	iota	I
κ	kappa	K
λ	lambda	Λ
μ	mu	M
ν	nu	N
ξ	xi	Ξ
o	omicron	O
π	pi	Π
ρ	rho	P
σ	sigma	Σ
τ	tau	T
υ	upsilon	Υ
ϕ	phi	Φ
χ	chi	X
ψ	psi	Ψ
ω	omega	Ω

02° Show that, for any (position) vectors x' and x'' in \mathbf{R}^n :

$$\langle x', x'' \rangle = \frac{1}{4}(\|x' + x''\|^2 - \|x' - x''\|^2)$$

One refers to this relation as the *Polarization Identity*.

03° Let x' and x'' be (position) vectors in \mathbf{R}^3 :

$$x' = \begin{pmatrix} u' \\ v' \\ w' \end{pmatrix}, \quad x'' = \begin{pmatrix} u'' \\ v'' \\ w'' \end{pmatrix}$$

and let x be the *cross product* of x' and x'' :

$$x = \begin{pmatrix} u \\ v \\ w \end{pmatrix} = x' \times x'' = \begin{pmatrix} v'w'' - w'v'' \\ w'u'' - u'w'' \\ u'v'' - v'u'' \end{pmatrix}$$

Verify that:

$$\langle\langle x', x' \times x'' \rangle\rangle = 0 \quad \text{and} \quad \langle\langle x'', x' \times x'' \rangle\rangle = 0$$

04° Find the angle θ between the (position) vectors:

$$x' = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \quad x'' = \frac{1}{2} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

05° Let P be the plane in \mathbf{R}^3 composed of all (position) vectors of the form:

$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

where s and t are any real numbers. Find a (nonzero) *direction* vector n :

$$n = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

and a real number d such that, for each (position) vector x in \mathbf{R}^3 , x lies in P iff:

$$\langle\langle x, n \rangle\rangle = d$$

06• Let a be a positive real number. Let d be the position vector:

$$d = \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix}$$

Let x be any (position) vector:

$$x = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

For each real number t , one may form the corresponding position vector s on the line joining d and x :

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = s = (1-t)d + tx = \begin{pmatrix} (1-t)a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} tu \\ tv \\ tw \end{pmatrix}$$

Find the value of t for which $p = 0$. In turn, solve for q and r in terms of a , u , v , and w . Note that the foregoing computation fails for certain position vectors x . In the lectures, we will show that the foregoing procedure provides an algorithm for **perspective drawing**. Show that lines in the *real world* perpendicular to the *picture plane* appear in the picture plane as lines converging to the *vanishing point*. What can be said of the various other lines in the real world?