On the RedShift in Universal Space

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1 Objective

01° In 1976, Irving Segal proposed a new model for time and space. He called it Universal Space. The model derives in part from the fact that Maxwell's Equations are not only Relativistically but also Conformally Invariant. It leads to novel and provocative reinterpretations of the RedShift and the Cosmic Background Radiation, which figure in modern cosmology. See [xx] and [yy].

02° In this essay, we will describe the basic structures underlying Segal's proposal and we will present a formal proof of his quadratic *redshift-distance* relation:

$$z = tan^2(\frac{d}{2})$$

2 Universal Space

Basic Definitions

03° In what follows, let us identify vectors, linear mappings, and (Hermitean) bilinear forms with the corresponding coordinate arrays, defined relative to the appropriate standard bases.

04° We begin with the following short exact sequence of separable, locally compact groups:

$$0 \longrightarrow \pi\mathbf{Z} \xrightarrow{\alpha} \mathbf{R}\times\mathbf{SU}(2) \xrightarrow{\beta} \mathbf{U}(2) \longrightarrow 1$$

The homomorphisms α and β are defined as follows:

$$\alpha(\pi k) = (\pi k, e^{-i\pi k}I), \qquad \beta(u, V) = e^{iu}V$$

where k is any integer, where u is any real number, and where V is any matrix in $\mathbf{SU}(2)$. Obviously, $\mathbf{R} \times \mathbf{SU}(2)$ is the simply connected covering group for $\mathbf{U}(2)$. We refer to it as Universal Space. We refer to $\mathbf{U}(2)$ as the Conformal Compactification of $\mathbf{R}^4$.

05° We may identify $\mathbf{R}^4$ with $\mathbf{H}(2)$, as follows. For each quadruple:

$$\epsilon = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$$

in $\mathbf{R}^4$, we introduce the matrix:

$$H = \begin{pmatrix} t+z & x-iy \\ x+iy & t-z \end{pmatrix}$$

in $\mathbf{H}(2)$. The mapping so defined is a linear isomorphism carrying $\mathbf{R}^4$ to $\mathbf{H}(2)$. The Standard Basis for $\mathbf{R}^4$ corresponds to the Pauli Basis for $\mathbf{H}(2)$:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Moreover:

$$t = \frac{1}{2}tr(H) \quad \text{and} \quad t^2 - x^2 - y^2 - z^2 = det(H)$$

In turn, we may embed $\mathbf{H}(2)$ in $\mathbf{U}(2)$ by the Cayley Transformation C. For each matrix H in $\mathbf{H}(2)$, we define the matrix:

$$C(H) = [I + (1/2)iH][I - (1/2)iH]^{-1} = [I - (1/2)iH]^{-1}[I + (1/2)iH]$$

in $\mathbf{U}(2)$. One can easily check that the mapping C so defined is injective and that its range consists of the matrices U in $\mathbf{U}(2)$ for which $I + U$ is invertible. Moreover, for each such matrix U in $\mathbf{U}(2)$, one may compute the matrix H in $\mathbf{H}(2)$ for which $C(H) = U$ as follows:

$$H = 2i(I - U)(I + U)^{-1} = 2i(I + U)^{-1}(I - U)$$

Finally, we may identify $\mathbf{S}^3$ with $\mathbf{SU}(2)$, as follows. For each quadruple:

$$q = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

in $\mathbf{S}^3$, we introduce the matrix:

$$V = \begin{pmatrix} a+ib & -c+id \\ c+id & a-ib \end{pmatrix}$$

in $\mathbf{SU}(2)$. The mapping so defined is a homeomorphism carrying $\mathbf{S}^3$ to $\mathbf{SU}(2)$. Consequently, we may identify Universal Space with $\mathbf{R} \times \mathbf{S}^3$.

06° Let us organize the foregoing mappings in the following diagram:

$$\begin{tikzcd}
 & & \mathbf{S}^3 \arrow[d, equal] & & \\
0 \arrow[r] & \pi\mathbf{Z} \arrow[r, "\alpha"] & \mathbf{R} \times \mathbf{SU}(2) \arrow[r, "\beta"] & \mathbf{U}(2) \arrow[r] & 1 \\
 & & & \mathbf{H}(2) \arrow[u, "C"'] \arrow[d, equal] & \\
 & & & \mathbf{R}^4 &
\end{tikzcd}$$

Conformal Structures

07° Let ϵ be any quadruple in $\mathbf{R}^4$. We may identify the tangent space $T_\epsilon(\mathbf{R}^4)$ of $\mathbf{R}^4$ at ϵ with $\mathbf{R}^4$ itself. The future cone in $T_\epsilon(\mathbf{R}^4)$ consists of the quadruples:

$$\bar{\epsilon} = \begin{pmatrix} \bar{t} \\ \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix}$$

such that $0 \leq \bar{t}$ and $0 \leq \bar{t}^2 - \bar{x}^2 - \bar{y}^2 - \bar{z}^2$. Such cones comprise the Conformal Structure for $\mathbf{R}^4$.

08° Let H be any matrix in $\mathbf{H}(2)$. We may identify the tangent space $T_H(\mathbf{H}(2))$ of $\mathbf{H}(2)$ at H with $\mathbf{H}(2)$ itself. Invoking the identification of $\mathbf{R}^4$ with $\mathbf{H}(2)$, we may introduce an appropriate future cone in $T_H(\mathbf{H}(2))$. It consists of the matrices F in $\mathbf{H}(2)$ such that $0 \leq F$. That is, $0 \leq tr(F)$ and $0 \leq det(F)$. Such cones comprise the Conformal Structure for $\mathbf{H}(2)$.

09° Finally, let U be any matrix in $\mathbf{U}(2)$. We may identify the tangent space $T_U(\mathbf{U}(2))$ of $\mathbf{U}(2)$ at U with $i\mathbf{H}(2)U$ and we may introduce an appropriate future cone in $T_U(\mathbf{U}(2))$, consisting of the matrices iGU in $\mathbf{iH}(2)U$ such that $0 \leq G$. Such cones comprise the Conformal Structure for $\mathbf{U}(2)$.

10° Granted the foregoing definitions of conformal structures on $\mathbf{H}(2)$ and $\mathbf{U}(2)$, let us prove that C is conformal. To that end, let H be any matrix in $\mathbf{H}(2)$ and let $U = C(H)$ be the corresponding matrix in $\mathbf{U}(2)$. For each s in $\mathbf{R}$ and for each matrix F in $\mathbf{H}(2)$, let us introduce the notation:

$$\begin{aligned} \lambda(s) &= I + (1/2)i(H + sF) \\ \mu(s) &= I - (1/2)i(H + sF) \end{aligned}$$

We have:

$$\begin{aligned} DC(H)(F) &= \frac{d}{ds}C(H + sF)\big|_{s=0} \\ &= \lim_{s\to 0}\frac{1}{s}(\mu(s)^{-1}\lambda(s) - \lambda(0)\mu(0)^{-1}) \\ &= \lim_{s\to 0}\frac{1}{s}(\mu(s)^{-1}[\lambda(s)\mu(0) - \mu(s)\lambda(0)]\mu(0)^{-1}) \\ &= \lim_{s\to 0}\frac{1}{s}(\mu(s)^{-1}[isF]\mu(0)^{-1}) \\ &= i\mu(0)^{-1}F\mu(0)^{-1} \\ &= i\mu(0)^{-1}F\lambda(0)^{-1}U \\ &= iGU \end{aligned}$$

where $G = (I - (1/2)iH)^{-1}F(I + (1/2)iH)^{-1}$. Obviously, if $0 \leq F$ then $0 \leq G$. Hence, C is conformal.

11° Let us emphasize that if $H = 0$ then $U = I$ and:

$$DC(0)(F) = iF$$

12° Of course, we may invoke the covering homomorphism β to induce a Conformal Structure on $\mathbf{R} \times \mathbf{SU}(2)$. Naturally, β is conformal.

Conformal Inversion

13° For later reference, let us introduce the mapping Q carrying $\mathbf{U}(2)$ to itself, defined as follows:

$$Q(U) = -\frac{1}{det(U)}U$$

where U is any matrix in $\mathbf{U}(2)$. Obviously, Q is involutory. Let us prove that Q is anti-conformal. To that end, let U be any matrix in $\mathbf{U}(2)$ and let $V = Q(U)$ be the corresponding matrix in $\mathbf{U}(2)$. For each matrix F in $\mathbf{H}(2)$, we have:

$$\begin{aligned} DQ(U)(iFU) = & \lim_{s\to 0} \frac{1}{s}[Q(e^{isF}U) - Q(U)] \\ = & -\lim_{s\to 0} \frac{1}{s}[Q(e^{isF})Q(U) + Q(U)] \\ = & -\lim_{s\to 0} \frac{1}{s}(Q(e^{isF}) - Q(I))Q(U) \\ = & \frac{d}{ds}[det(e^{-isF})e^{isF}]\big|_{s=0}Q(U) \\ = & iGV \end{aligned}$$

where $G = F - tr(F)I$. Obviously, if $0 \leq F$ then $G \leq 0$. Hence, Q is anti-conformal. One refers to Q as *conformal inversion* on $\mathbf{U}(2)$.

14° Let us apply C to interpret Q as a mapping carrying (part of) $\mathbf{H}(2)$ to itself. For preparation, we introduce the open dense subset $\mathbf{U}_-$ of $\mathbf{U}(2)$ consisting of all matrices U such that $I + U$ is invertible and the open dense subset $\mathbf{U}_+$ of $\mathbf{U}(2)$ consisting of all matrices V such that $I - V$ is invertible. Of course, $\mathbf{U}_-$ is the range of C. In turn, we introduce the intersection $\mathbf{U}_\pm = \mathbf{U}_- \cap \mathbf{U}_+$, an open dense subset of $\mathbf{U}(2)$. For any matrices U and V in $\mathbf{U}(2)$, we claim that $U \in \mathbf{U}_-$ iff $I + det(U)^{-1}U$ is invertible and that $V \in \mathbf{U}_+$ iff $I - det(V)^{-1}V$ is invertible. To prove the claim, we simply note that -1 is an eigenvalue for U iff $-det(U)$ is an eigenvalue for U, and that 1 is an eigenvalue for V iff $det(V)$ is an eigenvalue for V. Consequently, $Q(\mathbf{U}_-) = \mathbf{U}_+$ and $Q(\mathbf{U}_+) = \mathbf{U}_-$. Hence:

$$Q(\mathbf{U}_\pm) = \mathbf{U}_\pm$$

By article 5°, we find that the open dense subset:

$$\mathbf{H}_\pm = C^{-1}(\mathbf{U}_\pm)$$

of $\mathbf{H}(2)$ consists of the matrices H for which H is invertible. Let P be the mapping carrying $\mathbf{H}_\pm$ to itself, defined as follows:

$$P(H) = 4\frac{1}{det(H)}H$$

where H is any matrix in $\mathbf{H}_\pm$. We contend that:

$$C(P(H)) = Q(C(H))$$

One can easily check that the foregoing relation is invariant under conjugation by matrices W in $\mathbf{U}(2)$. Hence, to prove the relation, we need only consider invertible matrices H in $\mathbf{H}(2)$ which are diagonal. For such matrices, one can prove the relation by routine computation.

3 The Conformal Group

Basic Definitions

15° Let $\mathbf{\Gamma}$ be the connected component of the identity in the group consisting of all conformal transformations carrying $\mathbf{U}(2)$ to itself. One can show that $\mathbf{\Gamma}$ is a Lie group. Let $\hat{\mathbf{\Gamma}}$ be the simply connected covering group for $\mathbf{\Gamma}$. Of course, $\hat{\mathbf{\Gamma}}$ is a Lie group and it acts on the simply connected covering space $\mathbf{R} \times \mathbf{SU}(2)$ for $\mathbf{U}(2)$. We intend to describe $\mathbf{\Gamma}$ in terms of matrices M in $\mathbf{M}(4, \mathbf{C})$.

16° We begin by noting that $\mathbf{M}(4, \mathbf{C})$ acts (partially) on $\mathbf{M}(2, C)$. Thus, let M be any matrix in $\mathbf{M}(4, \mathbf{C})$:

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

In the foregoing expression and in the many others like it which follow, we intend that the matrix entries be themselves 2 by 2 (in general, complex) matrices. Let $dom(M)$ be the set of all matrices N in $\mathbf{M}(2, \mathbf{C})$ for which $CN + D$ is invertible. For any such matrix N, we define:

$$M.N = (AN + B)(CN + D)^{-1}$$

Obviously, for the identity matrix J in $\mathbf{M}(4, \mathbf{C})$:

$$J = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}$$

we have $dom(J) = \mathbf{M}(2, \mathbf{C})$ and, for any matrix N in $dom(J)$:

$$J.N = N$$

Moreover, for any matrices M_1 and M_2 in $\mathbf{M}(4, \mathbf{C})$:

$$M_1 = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix}, \quad M_2 = \begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix}$$

and for any matrix N in $\mathbf{M}(2,\mathbf{C})$, if $N \in dom(M_2)$ and $M_2.N \in dom(M_1)$, we have:

$$(M_1 M_2).N = M_1.(M_2.N)$$

To verify the foregoing relation, we compute:

$$\begin{aligned}
&[A_1(A_2N+B_2)(C_2N+D_2)^{-1}+B_1] \\
&\qquad\qquad \{C_1(A_2N+B_2)(C_2N+D_2)^{-1}+D_1\}^{-1} \\
&= [A_1(A_2N+B_2)+B_1(C_2N+D_2)][C_2N+D_2]^{-1} \\
&\qquad\qquad \{[C_1(A_2N+B_2)+D_1(C_2N+D_2)][C_2N+D_2]^{-1}\}^{-1} \\
&= [(A_1A_2+B_1C_2)N+(A_1B_2+B_1D_2)] \\
&\qquad\qquad [(C_1A_2+D_1C_2)N+(C_1B_2+D_1D_2)]^{-1}
\end{aligned}$$

Two Groups

17° Now let us describe two distinct but mutually conjugate subgroups of the group $\mathbf{GL}(4,\mathbf{C})$:

$$\mathbf{SU}(2,2)^\bullet, \quad \mathbf{SU}(2,2)^\circ$$

In terms of the (partial) action of $\mathbf{M}(4,\mathbf{C})$ on $\mathbf{M}(2,\mathbf{C})$ just defined, we will show that the group $\mathbf{SU}(2,2)^\bullet$ implements, in particular, the action of Lorentz Transformations on $\mathbf{H}(2)$ while the group $\mathbf{SU}(2,2)^\circ$ implements the action of Conformal Transformations on $\mathbf{U}(2)$. Moreover, we will show that there is a matrix K in $\mathbf{M}(4,\mathbf{C})$ which defines the relation of conjugacy between $\mathbf{SU}(2,2)^\bullet$ and $\mathbf{SU}(2,2)^\circ$ and which implements the Cayley Transformation C.

18° We begin the description by introducing two Hermitean bilinear forms on $\mathbf{C}^4$:

$$\beta^\bullet = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}, \qquad \beta^\circ = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

For any matrix M in $\mathbf{M}(4,\mathbf{C})$:

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

we may form the adjoint $M^\bullet$ of M relative to $\beta^\bullet$:

$$M^t \beta^\bullet = \beta^\bullet \overline{M^\bullet}$$

It turns out that:

$$M^\bullet = \begin{pmatrix} D^* & -B^* \\ -C^* & A^* \end{pmatrix}$$

Similarly, we may form the adjoint M° of M relative to β°:

$$M^t\beta^\circ = \beta^\circ\overline{M^\circ}$$

It turns out that:

$$M^\circ = \begin{pmatrix} A^* & -C^* \\ -B^* & D^* \end{pmatrix}$$

19° Now let $\mathbf{SU}(2,2)^\bullet$ be the subgroup of $\mathbf{GL}(4,\mathbf{C})$ consisting of all matrices M for which $det(M) = 1$ and for which the pullback of $\beta^\bullet$ by M is $\beta^\bullet$:

$$\beta^\bullet = M^t\beta^\bullet\bar{M}$$

The latter condition means that $M^{-1} = M^\bullet$. That is:

$$(\bullet) \qquad \begin{array}{ll} D^*A - B^*C = I, & D^*B - B^*D = 0 \\ A^*C - C^*A = 0, & A^*D - C^*B = I \end{array}$$

Similarly, let $\mathbf{SU}(2,2)^\circ$ be the subgroup of $\mathbf{GL}(4,\mathbf{C})$ consisting of all matrices M for which $det(M) = 1$ and for which the pullback of β° by M is β°:

$$\beta^\circ = M^t\beta^\circ\bar{M}$$

The latter condition means that $M^{-1} = M^\circ$. That is:

$$(\circ) \qquad \begin{array}{ll} A^*A - C^*C = I, & A^*B - C^*D = 0 \\ D^*C - B^*A = 0, & D^*D - B^*B = I \end{array}$$

Conjugacy

20° Let K be the following matrix in $\mathbf{GL}(4,\mathbf{C})$:

$$K = \begin{pmatrix} (1/2)iI & I \\ -(1/2)iI & I \end{pmatrix}$$

Obviously:

$$K^{-1} = \begin{pmatrix} -iI & iI \\ (1/2)I & (1/2)I \end{pmatrix}$$

One can easily check that the pullback of β° by K is $-i\beta^\bullet$:

$$-i\beta^\bullet = K^t\beta^\circ\bar{K}$$

Consequently, $\mathbf{SU}(2,2)^\bullet$ and $\mathbf{SU}(2,2)^\circ$ are conjugate in $\mathbf{GL}(4,\mathbf{C})$ under K:

$$K\,\mathbf{SU}(2,2)^\bullet K^{-1} = \mathbf{SU}(2,2)^\circ$$

21° Obviously, $\mathbf{H}(2) \subseteq dom(K)$ and $\mathbf{U}_- \subseteq dom(K^{-1})$. Moreover, for each matrix H in $\mathbf{H}(2)$:

$$K.H = [I + (1/2)iH][I - (1/2)iH]^{-1} = C(H)$$

and, for each matrix U in $\mathbf{U}_-$:

$$K^{-1}.U = 2i(I - U)(I + U)^{-1}$$

so that $C(K^{-1}.U) = U$. See article 5°.

Two Actions

22° We contend first that $\mathbf{SU}(2,2)^\circ$ acts on $\mathbf{U}(2)$. To prove the contention, let M be any matrix in $\mathbf{SU}(2,2)^\circ$:

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

and let U be any matrix in $\mathbf{U}(2)$. From relations ($\circ$), one can easily derive the basic relation:

$$(AU + B)^*(AU + B) = (CU + D)^*(CU + D)$$

Hence, for any vector z in $\mathbf{C}^2$, if $(CU + D)z = 0$ then $(AU + B)z = 0$, so that $(D^*CU + D^*D)z = 0$ and $(B^*AU + B^*B)z = 0$. In turn:

$$z = (D^*D - B^*B)z = -(D^*C - B^*A)Uz = 0$$

We infer that both $CU + D$ and $AU + B$ are invertible. Moreover, by the foregoing basic relation:

$$[(AU + B)(CU + D)^{-1}]^* = [(AU + B)(CU + D)^{-1}]^{-1}$$

These observations prove the contention.

23° Let us prove that the action of $\mathbf{SU}(2,2)^\circ$ on $\mathbf{U}(2)$ is transitive. To that end, let U and V be any matrices in $\mathbf{U}(2)$. Let $W = V^{-1}U$ and let c be a complex number for which $c^4 det(W) = 1$. Let M be the matrix in $\mathbf{SU}(2,2)^\circ$ defined as follows:

$$M = c\begin{pmatrix} I & 0 \\ 0 & W \end{pmatrix}$$

Obviously, $M.U = V$.

24° Now let us prove that the action of $\mathbf{SU}(2,2)^\circ$ on $\mathbf{U}(2)$ is conformal. To that end, let M be any matrix in $\mathbf{SU}(2,2)^\circ$:

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

Let U be any matrix in $\mathbf{U}(2)$ and let:

$$V = M.U = (AU+B)(CU+D)^{-1}$$

be the corresponding matrix in $\mathbf{U}(2)$. We note that, by relations ($\circ$):

$$\begin{aligned} I &= U^*A^*(AU+B) - U^*C^*(CU+D) \\ &= U^*A^*(AU+B) - U^*C^*V^*(AU+B) \\ &= U^*(A-VC)^*(AU+B) \end{aligned}$$

so that:

$$(A-VC)^* = U(AU+B)^{-1}$$

Let F be any matrix in $\mathbf{H}(2)$. We find that:

$$\begin{aligned} &\frac{d}{ds} M.(e^{isF}U)\big|_{s=0} \\ &\qquad = \frac{d}{ds}[A(e^{isF}U)+B][C(e^{isF}U)+D]^{-1}\big|_{s=0} \\ &\qquad = (AiFU)(CU+D)^{-1} - (AU+B)(CU+D)^{-1}(CiFU)(CU+D)^{-1} \\ &\qquad = i(A-VC)FU(CU+D)^{-1} \\ &\qquad = iGV \end{aligned}$$

where $G = (A-VC)FU(AU+B)^{-1}$. Consequently, if $0 \le F$ then $0 \le G$. We conclude that the transformation on $\mathbf{U}(2)$ implemented by M is conformal.

25° Are there conformal transformations on $\mathbf{U}(2)$ other than those implemented by $\mathbf{SU}(2,2)^\circ$?

26° We contend second that $\mathbf{SL}(2,\mathbf{C})$ can be identified as a subgroup of $\mathbf{SU}(2,2)^\bullet$ and that, so identified, it implements the action of Lorentz Transformations on $\mathbf{H}(2)$. To prove the contention, let A be any matrix in $\mathbf{SL}(2,\mathbf{C})$. Let M_A be the matrix in $\mathbf{GL}(4,\mathbf{C})$ defined as follows:

$$M_A = \begin{pmatrix} A & 0 \\ 0 & (A^*)^{-1} \end{pmatrix}$$

Obviously, M_A lies in $\mathbf{SU}(2,2)^{\bullet}$. Moreover, $\mathbf{H}(2) \subseteq dom(M_A)$ and, for each H in $\mathbf{H}(2)$:

$$M_A.H = AHA^*$$

By the foregoing relation, we recognize that M_A defines the generic (proper orthochronous) Lorentz Transformation on $\mathbf{H}(2)$. We infer that the matrix:

$$KM_AK^{-1}$$

in $\mathbf{SU}(2,2)^{\circ}$, by its action on $\mathbf{U}$, carries $\mathbf{U}_- = K(\mathbf{H}(2))$ to itself and defines an equivalent form of the foregoing (proper orthochronous) Lorentz Transformation.

27° We ought to develop the relation between $\mathbf{U}(2)$ and the space $\mathbf{K}(2)$ identified with the *light rays* in $\mathbf{R}^6$, defined by the quadratic form:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

which one may identify with the quotient space of:

$$\mathbf{S}^1 \times \mathbf{S}^3$$

by the equivalence relation:

$$(\mathbf{t}', \mathbf{u}') \equiv (\mathbf{t}'', \mathbf{u}'') \iff (\exists s \neq 0)(\mathbf{t}'' = s\mathbf{t}' \wedge \mathbf{u}'' = s\mathbf{u}')$$

4 The Photon

28°

29°

5 RedShift

30°

31°