MATHEMATICS 322
THE THEOREM OF GROBMANN AND HARTMANN

1° Let n be a positive integer. Let V be an open subset of $\mathbf{R}^n$ and let F be a vector field on V. Assume that $F(0) = 0$, so that 0 is a *critical point* for F. Let $H := DF(0)$ be the first order approxiation to F. Assume that the real parts of the eigenvalues of H are nonzero, so that the critical point 0 is *hyperbolic*. The Theorem of Grobmann and Hartmann asserts (in particular) that there exists a vector field G on $\mathbf{R}^n$ such that:

(1) near 0, F and G are equal; that is, there exists an open subset U' of $\mathbf{R}^n$ such that $0 \in U'$, $U' \subseteq V$ and, for any x in U', $F(x) = G(x)$;

(2) far from 0, G and H are equal, that is, there exists an open subset U'' of $\mathbf{R}^n$ such that $0 \in U''$ and, for any y in $\mathbf{R}^n \backslash U''$, $G(y) = H(y)$; as a result, G is *complete*, that is, the integral curves for G are defined for all time;

(3) G and H are *equivalent*; that is, there exists a homeomorphism T carrying $\mathbf{R}^n$ to itself such that, for any w in $\mathbf{R}^n$ and for any t in $\mathbf{R}$:

$$\gamma_v(t) = T(\delta_w(t))$$

where $v := T(w)$, where γ_v is the integral curve for G passing through v at time 0, and where δ_w is the integral curve for H passing through w at time 0. Of course:

$$\delta_w(t) = e^{tH} w$$

2° In the lectures, we will interpret this technical array of statements.