Econ 312

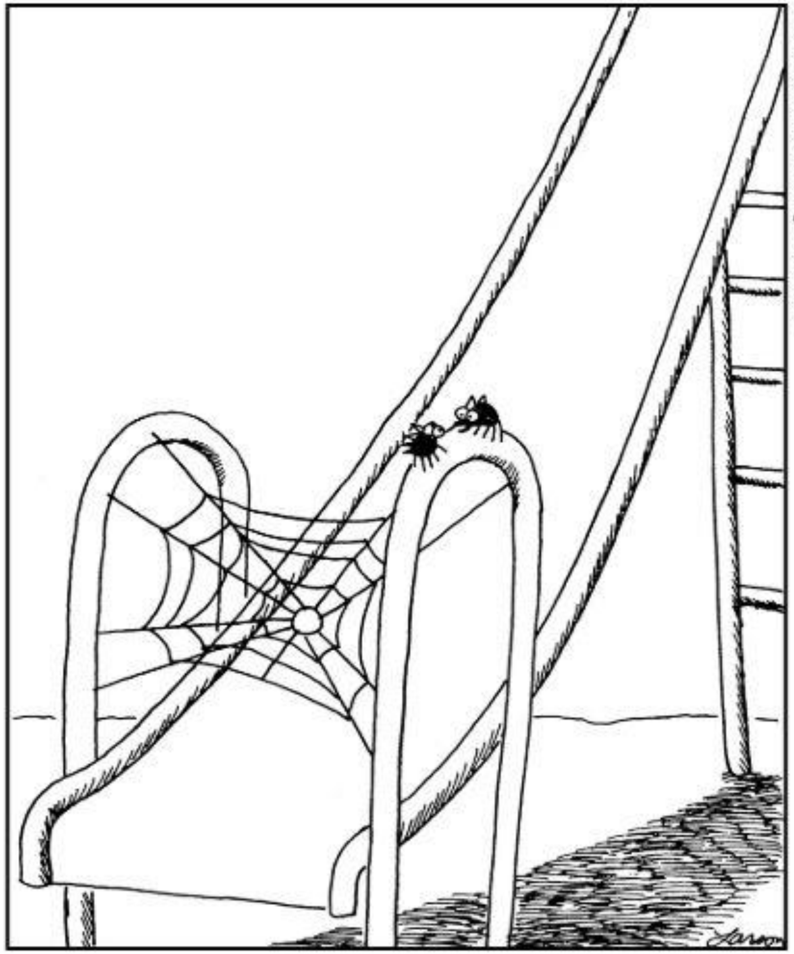
Wednesday, April 1
Linear Distributed Lag Models

Reading: Online time-series Chapter 3

Class notes: Pages 106 to 109

Daily problems: #26 AND #27

Today’s Far Side offering



“If we pull this off, we’ll eat like kings.”

Context and overview

- **Last class**: We examined the methods used for estimating models in which the error term is autocorrelated
- **Today**: We discuss how to estimate **distributed-lag** models in which the **deterministic** part of the equation involves linear, lagged effects
  - We consider what it means for a model to be **dynamically complete**
  - We look at **finite distributed lags** and the **Koyck lag** model
  - We extend these to consider the **ARX(*p*)** model and the **rational lag** model
  - Since theory never tells us about **lag length**, we consider alternative criteria for choosing the length of lags

Lagged effects

- In dynamic economic relationships a change in **x affects y slowly** over time, not all at once
- We can think of these effects in two equivalent ways:
  - The current y_t depends on current and past values $x_t, x_{t-1}, \ldots, x_{t-q}$
  - A change in the current value x_t affects both current and future $y_t, y_{t+1}, \ldots, y_{t+q}$
- We can think about modeling the effects of both:
  - **One-time changes** where x goes back to its original path in period $t + 1$
  - **Permanent changes** where x stays at its changed level in $t + 1$ and all future periods

Multipliers

- We measure lagged effects through **multipliers**:
  - **Impact (short-term) multiplier:** $\dfrac{\partial E(y_t)}{\partial x_t}$
  - ***s*-period-delay impact multiplier:** $\dfrac{\partial E(y_t)}{\partial x_{t-s}} = \dfrac{\partial E(y_{t+s})}{\partial x_t}$
  - **s-period cumulative multiplier:** $\sum_{i=0}^{s} \dfrac{\partial E(y_t)}{\partial x_{t-i}} = \sum_{i=0}^{s} \dfrac{\partial E(y_{t+i})}{\partial x_t}$
  - **Total (long-term) cumulative multiplier:** $\sum_{i=0}^{\infty} \dfrac{\partial E(y_t)}{\partial x_{t-i}} = \sum_{i=0}^{\infty} \dfrac{\partial E(y_{t+i})}{\partial x_t}$
- **Impact multipliers** measure effects of one-time changes
- **Cumulative multipliers** measure effects of permanent changes

Dynamically complete models

- Lag structure of x and the autocorrelation in u are closely related
- The more of the dynamic behavior of y we "soak up" in lags of x the less autocorrelation we usually find in u
- **Dynamically complete model** is one where the error term has no serial correlation (white noise) because **all** of the dynamics of y have been explained by lags in the deterministic model
- Now common to add lags until model is dynamically complete rather than correcting for autocorrelation via GLS estimation or Newey-West robust standard errors

Dynamically complete model for AR(1) error

$$y_t = \beta_0 + \beta_1 x_t + u_t,$$
$$u_t = \rho u_{t-1} + \varepsilon_t,$$
but $u_{t-1} = y_{t-1} - \beta_0 - \beta_1 x_{t-1},$

so $y_t = \beta_0 + \beta_1 x_t + \rho\left(y_{t-1} - \beta_0 - \beta_1 x_{t-1}\right) + \varepsilon_t,$ or

$$y_t = (1-\rho)\beta_0 + \rho y_{t-1} + \beta_1\left(x_t - \rho x_{t-1}\right) + \varepsilon_t.$$

We can write this as: $y_t = \gamma_0 + \gamma_1 y_{t-1} + \gamma_2 x_t + \gamma_3 x_{t-1} + \varepsilon_t$

- Adding a lag of *y* and a lag of *x* eliminates the AR(1) error
- Note that last equation has 4 coefficients that are functions of the 3 in the original model: We lose a degree of freedom

Finite distributed lags

- Most obvious choice: Put lagged x values on right-hand side:

$$y_t = \alpha + \beta_0 x_t + \beta_1 x_{t-1} + \ldots + \beta_q x_{t-q} + u_t = \alpha + \beta(L) x_t + u_t$$

- This is **finite distributed-lag model of order *q***

- Estimate by OLS
  - If q is large enough, should be able to make dynamically complete
  - That eliminates serial correlation of u

Multipliers in finite distributed lag model

- Impact (short-term) multiplier: $\frac{\partial E(y_t)}{\partial x_t} = \beta_0$
- *s*-period-delay impact multiplier: $\frac{\partial E(y_t)}{\partial x_{t-s}} = \frac{\partial E(y_{t+s})}{\partial x_t} = \beta_s$
- s-period cumulative multiplier: $\sum_{i=0}^{s} \frac{\partial E(y_t)}{\partial x_{t-i}} = \sum_{i=0}^{s} \frac{\partial E(y_{t+i})}{\partial x_t} = \sum_{i=0}^{s} \beta_i$
- Total (long-term) cumulative multiplier: $\sum_{i=0}^{\infty} \frac{\partial E(y_t)}{\partial x_{t-i}} = \sum_{i=0}^{\infty} \frac{\partial E(y_{t+i})}{\partial x_t} = \sum_{i=0}^{q} \beta_i$

- Effect of x on y **dies out after *q* periods**

Problems with finite distributed lag model

- **Multicollinearity** among lagged *x* terms
  - If *x* itself has high autocorrelation, then it x_t is highly correlated with x_{t-1}
  - This leads to very imprecise estimators so that the pattern of estimated lag **coefficients may not be smooth**: for example, β_1 could be positive, β_2 could be negative and then β_3 could be positive again, which is not intuitively plausible
  - The Koyck lag (next slide) is one example of an alternative that imposes smoothness on the lag coefficients
- **Determining *q***
  - Theory rarely tells us much about how long effects should last
- Long lags + short sample = **few degrees of freedom** left
  - You lose one observation at the beginning of the sample for each lag you add

Koyck lag: Treating *y* as an AR(1) process

- Add **lagged *y* as regressor** with current x:

$$y_t = \alpha + \phi_1 y_{t-1} + \beta_0 x_t + u_t$$

- Very **parsimonious** model with one parameter ϕ_1 handling all the lagged effects
- It **fits** many economic relationships well
- If gives **smoothly declining lagged effects**
- $|\phi_1|$ must be < 1 for stability
- Can estimate by OLS ***only if error is not serially correlated***

Multipliers in Koyck lag model

- Impact (short-term) multiplier: $\dfrac{\partial E(y_t)}{\partial x_t} = \beta_0$
- *s*-period-delay impact multiplier: $\dfrac{\partial E(y_t)}{\partial x_{t-s}} = \dfrac{\partial E(y_{t+s})}{\partial x_t} = \phi_1^s \beta_0$
- s-period cumulative multiplier: $\sum_{i=0}^{s} \dfrac{\partial E(y_t)}{\partial x_{t-i}} = \sum_{i=0}^{s} \dfrac{\partial E(y_{t+i})}{\partial x_t} = \beta_0 \sum_{i=0}^{s} \phi_1^i$
- Total (long-term) cumulative multiplier: $\sum_{i=0}^{\infty} \dfrac{\partial E(y_t)}{\partial x_{t-i}} = \sum_{i=0}^{\infty} \dfrac{\partial E(y_{t+i})}{\partial x_t} = \beta_0 \sum_{i=0}^{\infty} \phi_1^i = \dfrac{\beta_0}{1-\phi_1}$

- Effect of *x* on *y* **dies out slowly** with each period's effect being ϕ_1 times its predecessor
- This **smooth decay** of the effect often matches our intuition

Problems with the Koyck lag

- OLS is **biased and inconsistent** if there is serial correlation in u
  - This is the same problem as the Prais-Winsten estimator when there is a lagged dependent variable as a regressor
  - Can extend the model with additional lags of y and x to make it dynamically complete and eliminated serial correlation in u
- Pattern of decay in lagged effects is the same for all regressors if there is more than one x

Extending the Koyck model: ARX(p)

- Can add **more lagged *y*** to the equation:

$$y_t = \alpha + \phi_1 y_{t-1} + \ldots + \phi_p y_{t-p} + \beta_0 x_t + u_t,$$

$$\text{or } \phi(L) y_t = \alpha + \beta_0 x_t + u_t$$

- For stability, **roots of $\phi(L)$** must lie outside the unit circle
  - Effect dies out slowly, but with complex roots there may be cyclical convergence rather than simple exponential decay
- OLS is again biased and inconsistent if u is serially correlated
- Dynamic multipliers are determined by coefficients of $[\phi(L)]^{-1}$
- Again, decay pattern for all x will be the same

Combining Koyck with finite DL: ARDL(*p, q*)

- Adding **lags of *x*** to ARX(*p*) yields "rational lag" or ARDL model

$$y_t = \alpha + \phi_1 y_{t-1} + \ldots + \phi_p y_{t-p} + \beta_0 x_t + \beta_1 x_{t-1} + \ldots + \beta_q x_{t-q} + u_t$$

$$\text{or } \phi(L) y_t = \alpha + \beta(L) x_t + u_t,$$

$$y_t = \frac{\alpha}{\phi(L)} + \frac{\beta(L)}{\phi(L)} x_t + \frac{u_t}{\phi(L)}.$$

- Stable if roots of $\phi(L)$ lie outside the unit circle
- Estimate by OLS if no autocorrelation in u
- Dynamic multipliers are coefficients of (infinite) lag polynomial $\dfrac{\beta(L)}{\phi(L)}$

Choosing lag length

- How do we decide **how long the lags** (p and q above) should be?
- No uniform answer; analogous to deciding which variables to include or exclude as regressors
- We should nearly always **include all of the lags** between 1 and p and between 0 and q unless we have a good reason for omitting some of the intermediate lags
  - This is true even if they have insignificant coefficients

Criteria for choosing lag length

- Popular criteria:
  - Include lags with **significant coefficients**
  - Achieve **dynamic completeness** to eliminate serial correlation in error
  - Minimize Akaike and/or Schwartz/Bayesian **information criteria**
  - Maximize **adjusted *R*-square**
- **Important note**: Adding lags reduces sample size; be sure that the regressions compared with information criteria or adjusted *R*-square all have the **same set of observations**

Daily Problem #27

- In Daily Problem #27, you are given two tables of alternative estimates:
  - One with no lagged dependent variable and one with a lagged dependent variable
  - Both tables use the same set of observations and examine models with lag length 0 to 6
- Use the criteria discussed on the previous slide to decide which model you think is best. We will discuss this decision in the online class on Wednesday.

From *The Devil's Dictionary*

Grammar, *n*. A system of pitfalls thoughtfully prepared for the feet of the self-made [person], along the path by which he [or she] advances to distinction.

[I love arcane applications of grammar, such as "data" always being plural, the proper use of "that" and "which" for introducing subordinate clauses, and avoiding dangling prepositions and split infinitives. Future generations of thesis students will celebrate my passing from the ranks of first-draft readers.]

What's next?

- This session has examined incorporation of lagged effects of x on y into a regression model with stationary variables
- In the next class (April 3), we analyze **regressions with non-stationary variables**:
  - Testing variables for stationarity
  - Borderline stationary vs. non-stationary variables
- We also examine the phenomenon of **cointegration** and the **error-correction model** that is appropriate for them